Problem from Lecture 23. A router has four input links (I_0, I_1, I_2, and I_3) and four output links (O_0, O_1, O_2, and O_3). Each of the input and output ports has a buffer for four packets and uses first-come first-served (FCFS) packet scheduling. A crossbar connects the input and output ports.

The router operates in rounds. The speed of each input or output link is one packet per round. The speed of each crossbar bus is equal to two packets per round (i.e., the speedup is 2). When a packet starts arriving from an input link, the corresponding input port checks its buffer occupancy: if the buffer is full, the input port discards the packet; otherwise, the input port stores the arriving packet in the buffer. At the beginning of every round, the router: (a) configures the crossbar to transfer packets that have arrived to the input ports during previous rounds, and (b) frees the input buffer space occupied by packets that were transferred over the crossbar during the previous round. The router configures the crossbar in the increasing order of the input links, i.e., if two input ports I_k and I_j such that $k < j$ schedule packets for transfer to the same output port, the crossbar transfers first the packets from I_k. When a packet starts arriving from the crossbar, the corresponding output port checks its buffer occupancy: if the buffer is full, the output port discards the packet; otherwise, the output port stores the arriving packet in the buffer. At the beginning of every round, each output port: (a) starts transmitting a packet to its output link (unless the output port buffer is empty), and (b) frees the buffer space occupied by a packet that was transmitted to the link during the previous round.

Originally, the router contains no packets. By the end of each round 0 through 3, every input link delivers one packet to the router (i.e., sixteen packets overall). The first two packets from I_0 are destined for O_2. The last two packets from I_0 as well as all the packets from the other input links are destined for O_1.

Trace each packet. When are the packets transmitted to their output links or discarded? Explain your reasoning (it might be useful to track the buffer occupancies during each round).

Solution: *Let p_k^t be the packet that input link I_k delivers to the router by the end of round t.*

Round	0	1	2	3	4	5	6	7	8	9	10
Arrived	$p_3^0\ p_2^0\ p_1^0\ p_0^0$	$p_3^1\ p_2^1\ p_1^1\ p_0^1$	$p_3^2\ p_2^2\ p_1^2\ p_0^2$	$p_3^3\ p_2^3\ p_1^3\ p_0^3$							
I_0 buffer	p_0^0	$p_0^1\ p_0^0$	$p_0^2\ p_0^1$	$p_0^3\ p_0^2$	p_0^3						
I_1 buffer	p_1^0	$p_1^1\ p_1^0$	$p_1^2\ p_1^1$	$p_1^3\ p_1^2$	p_1^3						
I_2 buffer	p_2^0	$p_2^1\ p_2^0$	$p_2^2\ p_2^1$	$p_2^3\ p_2^2$	$p_2^3\ p_2^2$	$p_2^3\ p_2^2$					
I_3 buffer	p_3^0	$p_3^1\ p_3^0$	$p_3^2\ p_3^1\ p_3^0$	$p_3^3\ p_3^2\ p_3^1\ p_3^0$	$p_3^3\ p_3^2\ p_3^1\ p_3^0$	$p_3^3\ p_3^2\ p_3^1\ p_3^0$	$p_3^3\ p_3^2\ p_3^1\ p_3^0$	$p_3^3\ p_3^2$			
Crossbar		$p_2^0\ p_1^0\ p_0^0$	$p_2^1\ p_1^1\ p_0^1$	$p_1^2\ p_0^2$	$p_1^3\ p_0^3$	$p_2^3\ p_2^2$	$p_3^1\ p_3^0$	$p_3^3\ p_3^2$			
O_1 buffer		$p_2^0\ p_1^0$	$p_2^1\ p_1^1\ p_2^0\ p_1^0$	$p_0^2\ p_2^1\ p_1^1\ p_2^0$	$p_0^3\ p_0^2\ p_2^1\ p_1^1$	$p_2^2\ p_0^3\ p_0^2\ p_2^1$	$p_3^0\ p_2^2\ p_0^3\ p_0^2$	$p_3^2\ p_3^0\ p_2^2\ p_0^3$	$p_3^2\ p_3^0\ p_2^2$	$p_3^2\ p_3^0$	p_3^2
O_2 buffer		p_0^0	$p_0^1\ p_0^0$	p_0^1							
Transmitted			$p_1^0\ p_0^0$	$p_2^0\ p_0^1$	p_1^1	p_2^1	p_0^2	p_0^3	p_2^2	p_3^0	p_3^2
Discarded				p_1^2	p_1^3	p_2^3	p_3^1	p_3^3			