CoE/EE460
Switching Theory

Lecture 15

Washington University
Spring 2001

http://www.arl.wustl.edu/~lockwood/class/coe460/

Reading

- Topics
  - Technology mapping
  - Chortle Algorithm
- Additional Reading
  - John W. Lockwood, "Logic Synthesis for Field Programmable Gate Arrays", in The VLSI Handbook, IEEE/CRC Press, 2000, pp. 65.1-65.17.
    - Handouts provided in class

Chortle

- Designed for LUT-based FPGAs
    - Developed by:
        - Robert J. Francis, Jonathan Rose, Kevin Chung, Z. Vranesic
        - University of Toronto (1990s)
- Input: Optimized SOP Expression in Tree (T)
    - Internal nodes (n) implemented as AND or OR gates
    - Edges represent signals or inverted signals
- Method of operation: Dynamic programming
    - Compute & record optimal solution of sub-problems
    - MinMap(n,U_n) for $U_n \in$ a2..K
- Output:
    - Implementation of Circuit with fewest K-input LUTs

Chortle Terminology

- Post-Order Tree Traversal
    - Visit Left subtree
    - Visit Right subtree
    - Visit Root
- Utilization of LUT at root of subtree
    - $U \in \{ 2..K \}$
    - U=K for fully utilized LUT
- Minimum Cost of sub-circuit rooted at n
    - MinMap(n,U_n)
    - Optimal solution for Node n for $U \in \{ 2..K \}$
    - Note: MinMap(n,1) = LUT + MinMap(n,4)
- Utilization Division of Root Subtree
    - μ(# of inputs from left , # of inputs from right)

Chortle Algorithm

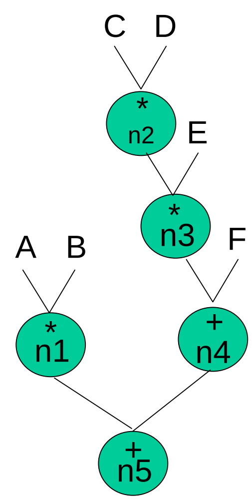
- MapTree(T,K)
  - For each node (n) in postorder traversal of Tree (T)
    - For each utilization (U) = 2 to K of node n
      - CurrentBestCost = ∞
      - CurrentBestMap = ∅
      - For each Utilization Divisions (μ(left,right)) such that left+right=U
        - » Construct minimum-cost mapping, M, for subtree rooted at n
        - » Calculate cost(M)
        - » If Cost(M) < CurrentBestCost(M)

          CurrentBestMap = M

          CurrentBestCost = Cost(M)
      - MinMap(n,U) = CurrentBestMap
  - Return MinMap(root,K)

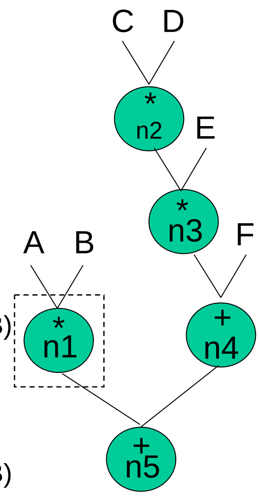
Chortle Example (Showing steps)

- Given:
  - F = A*B + (C*D*E) + F
- Decompose
  - F = (A*B) + (C*D) * E) + F
- Given
  - K=4
- Find Optimal Implementation
  - Maptree(n5,2)
  - Maptree(n5,3)
  - Maptree(n5,4)



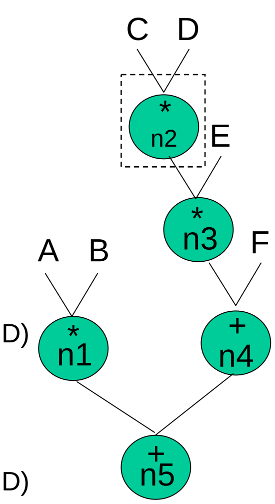
Chortle Example (Showing steps)

– For n=n1
- For U=2
  - For $\mu_{n1}(1,1)$
    - » CurrentBestMap=n1(A,B)
    - » CurrentBestCost(M) = 1 LUT
  - MinMap(n1,2) = n1(A,B)
    - » MinCost = 1 LUT
- For U=3 : Same as U=2
  - MinMap(n1,3) = MinMap(n1,2)=n1(A,B)
    - » MinCost = 1 LUT
- For U=4 : Same as U=2
  - MinMap(n1,4) = MinMap(n1,2)=n1(A,B)
    - » MinCost = 1 LUT



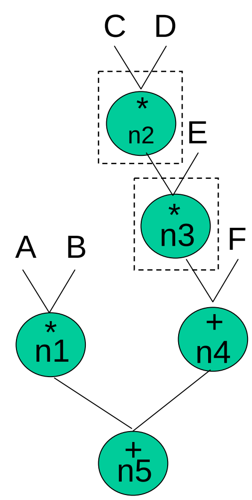
Chortle Example (Showing steps)

– For n=n2
- For U=2
  - For $\mu_{n2}(1,1)$
    - » CurrentBestMap=n2(C,D)
    - » CurrentBestCost(M) = 1 LUT
  - MinMap(n2,2) = n2(C,D)
    - » MinCost = 1 LUT
- For U=3 : Same as U=2
  - MinMap(n2,3) = MinMap(n2,2)=n2(C,D)
    - » MinCost = 1 LUT
- For U=4 : Same as U=2
  - MinMap(n2,4) = MinMap(n2,2)=n2(C,D)
    - » MinCost = 1 LUT



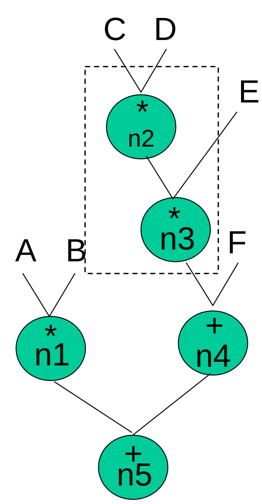
Chortle Example [continued]

- For n=n3
  - For U=2
    - For $\mu_{n3}(1,1)$
      - » CurrentBestMap
      - » =n3(MinMap(n2,K=4),E)
      - » =n3(n2(C,D),E)
      - » CurrentBestCost(M) =
      - » =MinCost(n2) + 1
      - » =2 LUTs
    - MinMap(n3,2) = n3(n2(C,D),E)
      - » MinCost = 2 LUTs



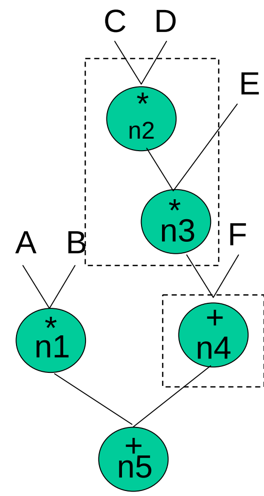
Chortle Example [continued]

- For n=n3 (continued)
  - For U=3
    - For $\mu_{n3}(2,1)$
      - » CurrentBestMap
      - » =n3(MinMap(n2,2),E)
      - » =n3(C,D,E)
      - » CurrentBestCost(M) = 1 LUT
    - For: $\mu_{n3}(1,2)$: Same as $\mu_{n3}(1,1)$
    - MinMap(n3,3) = n3(C,D,E)
      - » MinCost = 1 LUT
  - For U=4 : Same as U=3



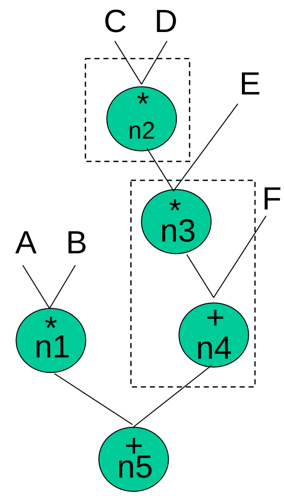
Chortle Example [continued]

- For n=n4
  - For U=2
    - For μ(left,right)=μ(1,1)
      - » CurrentBestMap
      - » =n4(MinMap(n3,K=4),F)
      - » =n4(n3(C,D,E),F)
      - » CurrentBestCost(M) = 2 LUTs
    - MinMap(n4,2) = n4(n3(C,D,E),F)
      - » MinCost = 2 LUTs



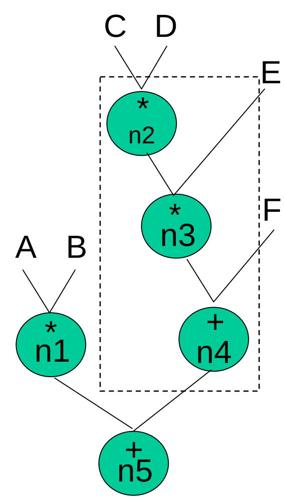
Chortle Example [continued]

- For n=n4 (continued)
  - For U=3
    - For $\mu_{n4}(2,1)$:
      - » CurrentBestMap
      - » =n4(MinMax(n3,2),F)
      - » =n4(n2(C,D),E,F)
      - » CurrentBestCost(M) = 2 LUTs
    - For: $\mu_{n4}(1,2)$: Same as $\mu_{n4}(1,1)$
      - » Cost = 2 LUTs (tie)
    - MinMap(n4,3) = n4(n2(C,D),E,F)
      - » MinCost = 2 LUTs



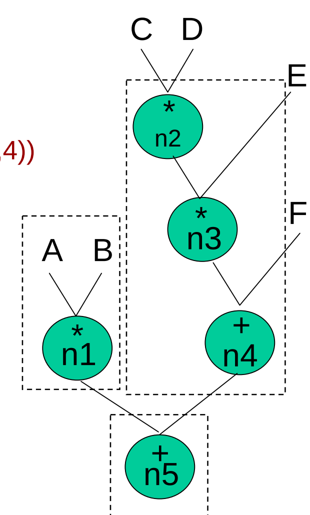
Chortle Example [continued]

– For n=n4 (continued)
- For U=4
  - – For μ(left,right)=μ(3,1)
    - » CurrentBestMap
    - » =n4(MinMap(n3,3),F)
    - » =n4(n3(C,D,E),F)
    - » CurrentBestCost(M) = 1 LUT
  - – MinMap(n4,4) = n4(C,D,E,F)
    - » MinCost = 1 LUT



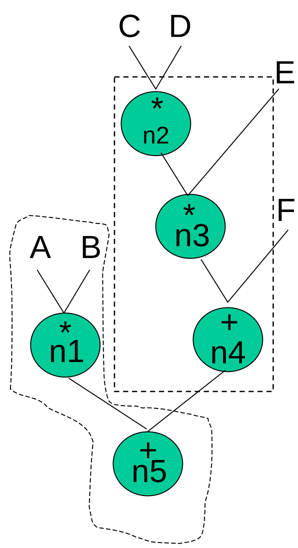
Chortle Example [continued]

– For n=n5
- For U=2
  - – For μ_{n5}=μ(1,1)
    - » CurrentBestMap
    - » =n5(MinMap(n1,4),MinMap(n4,4))
    - » =n5(n1(A,B),n4(C,D,E,F))
    - » CurrentBestCost(M)
    - » = 1+1+1=3
  - – MinMap(n5,2)
  - – = n5(n1(A,B),n4(C,D,E,F))
    - » MinCost = 3 LUTs



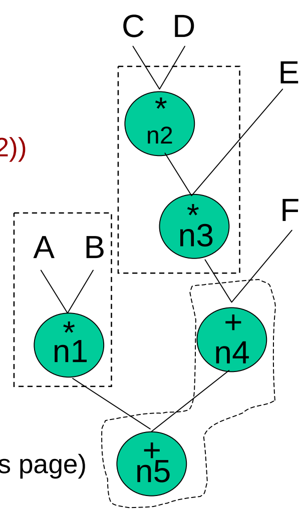
Chortle Example [continued]

– For n=n5 (continued)
  - For U=3
    – For $\mu_{n5}(2,1)$
      » CurrentBestMap
      » =n5(MinMap(n1,2),n4)
      » =n5(A,B,n4(C,D,E,F))
      » CurrentBestCost(M) =1+1
      » = 2 LUTs



Chortle Example [continued]

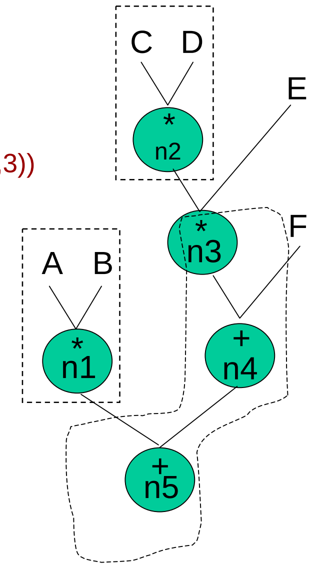
– For n=n5 (continued)
  - For U=3 (continued)
    – For $\mu_{n5}(1,2)$
      » CurrentBestMap
      » =n5(MinMap(n1,4),MinMap(n4,2))
      » = n5(n1(A,B),n3(C,D,E),F)
      » CurrentBestCost(M) = 1+1+1
      » =3 LUTS



– MinMap(n5,3)
– Comes from $\mu_{n5}(2,1)$ (From Previous page)
– = n5(A,B,n4(C,D,E,F))
  » MinCost = 2 LUTs

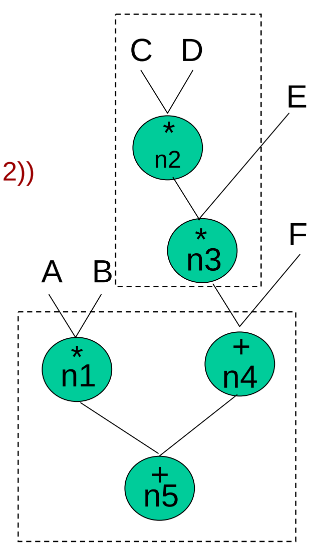
Chortle Example [continued]

- For n=n5 (continued)
  - For U=4
    - For $\mu_{n5}(1,3)$
      - » CurrentBestMap
      - » =n5(MinMap(n1,4),MinMap(n4,3))
      - » = n5(n1(A,B), n2(C,D),E,F)
      - » CurrentBestCost(M) = 1+1+1
      - » =3 LUTS



Chortle Example [continued]

- For n=n5 (continued)
  - For U=4 (continued)
    - For $\mu_{n5}(2,2)$
      - » CurrentBestMap
      - » =n5(MinMap(n1,2),MinMap(n4,2))
      - » = n5(A,B, n3(C,D,E), F)
      - » CurrentBestCost(M) = 1+1
      - » =2 LUTS
    - For $\mu_{n5}(3,1)$: Same as $\mu_{n5}(2,1)$
    - MinMap(n5,4)
    - = n5(A,B,n3(C,D,E),F)
      - » MinCost = 2 LUTs



Chortle : Optimal Solutions