CoE/EE460
Switching Theory

Lecture 18

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http://www.arl.wustl.edu/~lockwood/class/coe460/

Minimization of DFAs

- Consider a Moore Machine
- Initial Split Based on Output
  - P_1 = { S_1={C,D,E,F} , S_2={A,B,G,H} }
- Consider Successors
  - $P_{1.1}$ from S_1={C,D,E,F}
    - Given x=0:
      - S_{i+1} = { A, B, B, A }
      - S_{i+1} = { A:2, B:2, B:2, A:2 }
      - Cannot differentiate!
    - Given x=1:
      - S_{i+1} = { F, E, D, C }
      - S_{i+1} = { F:1, E:1, D:1, C:1 }
      - Cannot differentiate!

Present State	Next state w/ X=0	Next state w/ X=1	Output
A	C	G	0
B	D	H	0
C	A	F	1
D	B	E	1
E	B	D	1
F	A	C	1
G	D	D	0
H	C	C	0

Minimization of DFAs - Continued [$P_{1.2}$]

- Initial Split Based on Output
  - P_1 = { S_1={C,D,E,F} , S_2={A,B,G,H} }
- Consider Successors
  - $P_{2.1}$ from S_2={A,B,G,H}
    - Given x=0:
      - S_{i+1} = { C, D, D, C }
      - S_{i+1} = { C:1, D:1, D:1, C:1 }
      - Cannot differentiate!
    - Given x=1:
      - S_{i+1} = { G, H, D, C }
      - S_{i+1} = { G:2, H:2, D:1, C:1 }
      - Separate {A,B} , {G, H}

Pre-sent State	Next state w/ X=0	Next state w/ X=1	Output
A	C	G	0
B	D	H	0
C	A	F	1
D	B	E	1
E	B	D	1
F	A	C	1
G	D	D	0
H	C	C	0

Minimization of DFAs : Continued [$P_{2.1}$]

- Initial Split Based on Output
  - P_2 = { S_1={C,D,E,F} ,
    S_2={G,H},
    S_3={A,B} }
- Consider Successors
  - $P_{2.1}$ from S_1={C,D,E,F}
    - Given x=0:
      - S_{i+1} = { A, B, B, A }
      - S_{i+1} = { A:3, B:3, B:3, A:3 }
      - Cannot differentiate!
    - Given x=1:
      - S_{i+1} = { F, E, D, C }
      - S_{i+1} = { F:1, E:1, D:1, C:1 }
      - Cannot differentiate!

Pre-sent State	Next state w/ X=0	Next state w/ X=1	Output
A	C	G	0
B	D	H	0
C	A	F	1
D	B	E	1
E	B	D	1
F	A	C	1
G	D	D	0
H	C	C	0

Minimization of DFAs : Continued [$P_{2.2}$]

- Initial Split Based on Output
  - P_2 = { S_1={C,D,E,F} ,
    S_2={G,H},
    S_3={A,B} }
- Consider Successors
  - $P_{2.2}$ from S_2= {G,H}
    - Given x=0:
      - S_{i+1} = { D, C }
      - S_{i+1} = { D:1, C:1 }
      - Cannot differentiate!
    - Given x=1:
      - S_{i+1} = { D, C }
      - S_{i+1} = { D:1, C:1 }
      - Cannot differentiate!

Pre-sent State	Next state w/ X=0	Next state w/ X=1	Output
A	C	G	0
B	D	H	0
C	A	F	1
D	B	E	1
E	B	D	1
F	A	C	1
G	D	D	0
H	C	C	0

Minimization of DFAs : Continued [$P_{2.3}$]

- Initial Split Based on Output
  - P_2 = { S_1={C,D,E,F} ,
    S_2={G,H},
    S_3={A,B} }
- Consider Successors
  - $P_{2.3}$ from S_3= {A,B}
    - Given x=0:
      - S_{i+1} = { C, D }
      - S_{i+1} = { C:1, D:1 }
      - Cannot differentiate!
    - Given x=1:
      - S_{i+1} = { G, H }
      - S_{i+1} = { G:2, H:2 }
      - Cannot differentiate!

Pre-sent State	Next state w/ X=0	Next state w/ X=1	Output
A	C	G	0
B	D	H	0
C	A	F	1
D	B	E	1
E	B	D	1
F	A	C	1
G	D	D	0
H	C	C	0

Minimization of DFAs : Solution

- Initial Split Based on Output
  - P_3 = { S_1={C,D,E,F} ,
    S_2={G,H},
    S_3={A,B} }

- Therefore, DFA reduces to:

Present State	Next state w/ X=0	Next state w/ X=1	Output
S_3={A,B}	S_1	S_2	0
S_1={C,D,E,F}	S_3	S_1	1
S_2={G,H},	S_1	S_1	0

Pre-sent State	Next state w/ X=0	Next state w/ X=1	Output
A	C	G	0
B	D	H	0
C	A	F	1
D	B	E	1
E	B	D	1
F	A	C	1
G	D	D	0
H	C	C	0