CoE/EE460
Switching Theory

Lecture 2

Washington University
Spring 2001

http://www.arl.wustl.edu/~lockwood/class/coe460/

Announcements

- Office Hours:
  – Professor Lockwood
    - Office Hours: Tue 4pm-5pm
    - Location: Bryan 512 [Enter through Bryan 522]
  – Teaching Assistant: Henry Fu
    - Office Hours: Tue/Thr 3pm-4pm
    - Location: Bryan 414
- Reading
  – Hachtel: Chapter 3 - Boolean Algebras

Sets

- Set = Collection of objects, with { members }
  - Example
    - CPUs = { Pentium, K6, Athlon, MIPs, PowerPC }
    - X86_CPUs = { Pentium, K6, Athlon }
      - Defines a set of 80x86 processors
  - Example
    - Valid_Coordinates = { (x,y) : $x \in R$, $y \in R$, $x \geq y$ }
      - Defines a set that includes: { (3,2) , (6,3) , … }
  - Null = { } = $\varnothing$
- | Cardinality |
  - Number of elements in set

Sets (continued)

- Assuming:
    - CPUs = { Pentium, K6, Athlon, MIPs, 68K, PowerPC }
    - X86_CPUs = { Pentium, K6, Athlon }
    - MAC_CPUs = { 68K, PowerPC }
- Inclusion: $\subseteq$
  - $A \subseteq B$: True iff all members of A also in B
- Proper Inclusion: $\subset$
  - $A \subset B$: True iff $A \subseteq B$ and $A \neq B$
- Complement: ‘
  - A’ : Defines set of elements NOT in A
  - Must be specified for a universe, like U = CPUs

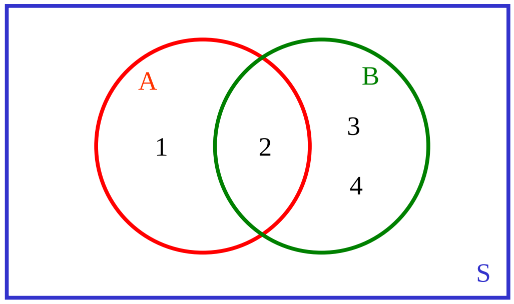
Sets (continued)

- Example
  - A = { 1,5,2}
  - B = { 5,7}
- Intersection: $\cap$
  - $A \cap B$: Elements in both A and B
- Union: $\cup$
  - $A \cup B$: Elements in either A or B
- Cartesian Product: $\times$
  - $A \times B$ = Ordered Pairs
  - $B \times A$ = Ordered Pairs

Examples

- Given
  - A={a,b,c}
  - B={a,b}
  - C={b,c,d}
  - D={b}
  - E={c,d}
- Evaluate (True/False)
  - $B \subseteq C$?
  - $B \subset B$?
  - $C \neq D$?
  - $D \cap E = \varnothing$ (disjoint) ?
  - $B \cap C = D$?

Venn Diagrams



- A = { ? }, |A|= ?
- B = { ? }, |B| = ?
- A∩B = { ? }, |A∩B|= ?
- A∪B = {   }, |A∪B|=|A|+|B|-|A∩B| = ?

2-Variable K-Map

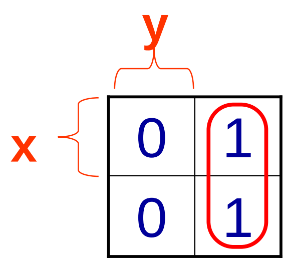
F (x,y)

y \ x	0	1
0	0	1
1	0	1

Minimization with 2-variable K-Map

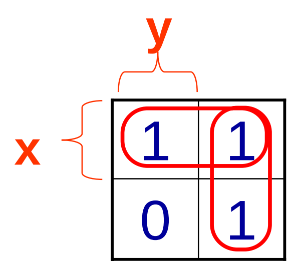
W = x'y' + xy'

W = y'(x' + x)
W = y'(1)
W = y'

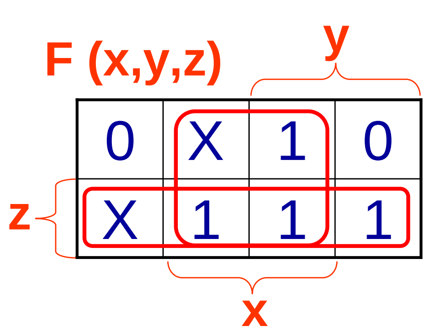


Z = xy + xy' + x'y'

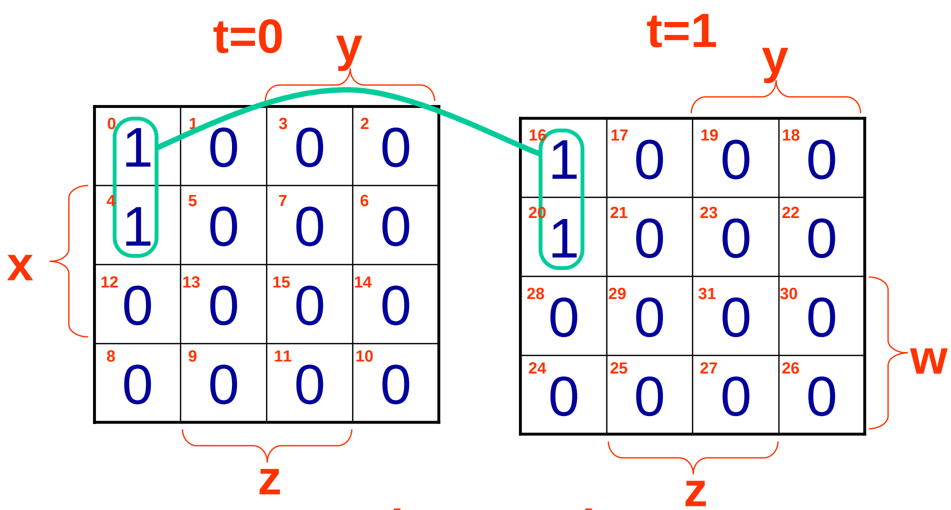
Z = (xy + xy') + (xy' + x'y')
Z = x(y+y') + y'(x+x')
Z = x(1) + y'(1)
Z = x + y'



3-Variable K-Map



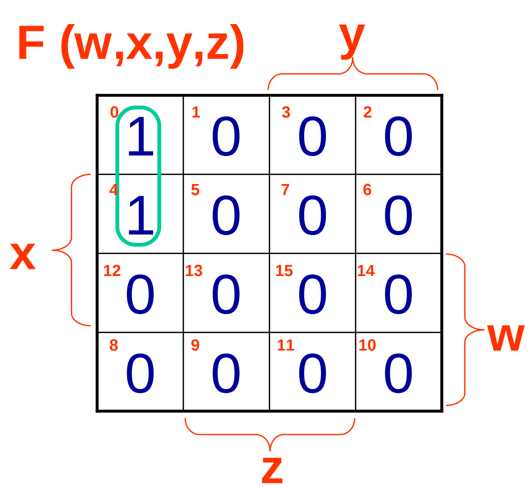
5-Variable K-Maps



F (t,w,x,y,z)

K-Maps

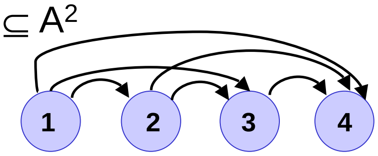
F (w,x,y,z)



Relations

- $\Re$ = Relation between set A and B
  - Subset of $A \times B$
  - $x \Re y \Leftrightarrow (x,y) \in \Re \subseteq A \times B$
- Consider
  - Relation $\leq$
  - Set A={1,2,3,4}
  - A^2 ={ (1,1),(1,2), … (4,4) }
  - $\Re \subseteq A^2$

	1	2	3	4
1	≤	≤	≤	≤
2		≤	≤	≤
3			≤	≤
4				≤



Relations

- Reflexivity
  - For every $x \in A$, $x \Re x$
    - $(x,x) \in \Re$
    - Relation exists in all elements of diagonal
- Symmetry
  - For every $(x,y) \in \Re$, $(y,x) \in \Re$
    - (1,2) is not matched by (2,1)
- Antisymmetry
  - If both $(x,y) \in \Re$ and $(y,x) \in \Re$ then x=y
    - (2,2) and (2,2) have x=y
    - (x,y) but no (y,x) for all $x \neq y$
    - Triangular matrix is Antisymetric

Relations (continued)

- Transitive
  - For every x,y,z
    - if $(x,y) \in \Re$ and $(y,z) \in \Re$ then $(x,z) \in \Re$
    - Example: $(1 \leq 2)$ $(2 \leq 3)$ implies $(1 \leq 3)$