CoE/EE460
Switching Theory

Lecture 4

Washington University
Spring 2001

http://www.arl.wustl.edu/~lockwood/class/coe460/

Announcements

- Solutions to HW1 on website

- HW2 Assignment
  - Out on Wednesday (2/7),
  - Due on Wednesday (2/14)

Boolean Function

- Function
  - Mapping of $f(x_1,x_2,..,x_n)$ to B, for each value of x_i
    - n : Number of variables
    - B : Set of
    - |B| : Cardinality of set
    - $2^n \log_2|B|$ elements
- Formula (Formulae)
  - Strings that contain variables and/or operations
    - Example: $x_1+x'_1x_2$

Boole's Expansion Theorem

- $F(x_1,x_2,..,x_n) = x_1' \bullet f(0, x_2,..,x_n) + x_1 \bullet f(1, x_2,..,x_n)$
  - Example:

$F = x,x_2+ x_1x_3$

$F = x_1' \bullet (0 \bullet x_2 + 0 \bullet x_3)$

$+ x_1 \bullet (1 \bullet x_2 + 1 \bullet x_3)$

$F = x_1' \bullet (0)$

$+ x_1 (x_2 + x_3)$

Example (In class)

$F_2 = x_1x_3'+ x_1 x_2 x_3$

$F_3 = x_2+ x'_1 x_2'x_3$

Minterm Canonical Form

- Recursively Apply Boole's Expansion
  - $F(x_1,x_2,..,x_n) = f(0,0,..,0)\ x_1' \bullet x_2' \bullet .. \bullet x_n'$
    $= f(1,0,..,0)\ x_1 \bullet x_2' \bullet .. \bullet x_n'$
    $= f(0,1,..,0)\ x_1' \bullet x_2 \bullet .. \bullet x_n'$
    $= f(1,1,..,0)\ x_1 \bullet x_2 \bullet .. \bullet x_n'$
    $= f(0,0,..,1)\ x_1' \bullet x_2' \bullet .. \bullet x_n$
    $= f(1,0,..,1)\ x_1 \bullet x_2' \bullet .. \bullet x_n$
    $= f(0,1,..,1)\ x_1' \bullet x_2 \bullet .. \bullet x_n$
    $= f(1,1,..,1)\ x_1 \bullet x_2 \bullet .. \bullet x_n$
  - $f(0,0,..,0)\ ..\ f(1,1,..,1)$ = discriminants
  - $x_1' \bullet x_2' \bullet .. \bullet x_n'\ ..\ x_1 \bullet x_2 \bullet .. \bullet x_n$ = minterms

Minterm Canonical Form

Example (In class)

Recursively apply Boole's theorem to find minterms for:

$F_2 = x_1x_3' + x_1 x_2 x_3$
$F_3 = x_2 + x'_1 x_2'x_3$

Maxterms

$F = [x_1' + f(1, x_2,..,x_n)] \bullet [x_1 + f(0, x_2,..,x_n)]$

– Same logic applies

Pseudo-Boolean Functions

- Consider
  - B={0,a,b,1}
  - Mappings
    - $F(0) \rightarrow a$
    - $F(a) \rightarrow a$
    - $F(b) \rightarrow b$
    - $F(1) \rightarrow 1$
- Minterms cononical form:
    - $F(x) = ax' + x = a + x$
- But:
    - For F(b), a+b=1, not b
- Therefore, F(x) is not a boolean function

Atoms of Boolean Algebra

- Consider minterms of f(x,y)
- Draw with
  - All 4 minterms
    - x'y' + x'y+xy' + xy
  - 3 minterms
    - x'y' + x'y + xy' +
    - x'y' + x'y + + xy
    - x'y' + + xy' + xy
    - + x'y + xy' + xy
  - 2 minterms
  - 1 minterms
  - No minterms

$$\binom{n}{k} = \frac{n!}{(k!)(n-k)!}$$

$$\binom{4}{4} = \frac{4!}{(4!)(4-4)!} = \frac{4!}{4!\cdot 1} = 1$$

$$\binom{4}{3} = \frac{4!}{(3!)(4-3)!} = \frac{4!}{3!} = 4$$

$$\binom{4}{2} = \frac{4!}{(2!)(4-2)!} = \frac{4!}{2!2!} = \frac{4\cdot 3\cdot 2}{2\cdot 2} = 6$$

$$\binom{4}{1} = \frac{4!}{(1!)(4-1)!} = \frac{4!}{3!} = 4$$

$$\binom{4}{0} = \frac{4!}{(1)(4)!} = 1$$