CoE/EE460
Switching Theory

Lectures 6

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Implicants

- Implicant
  - Product term included in f
  - Example: {xy , yz , xyz , xyz' , x'yz} in f=xy+yz
- Prime Implicant
  - Implicant not included in other implicant
  - Example: { xy , yz },
    - But not { xyz } because {xyz} $\in$ { yz }
- Essential Prime Implicant
  - Term includes minterm not otherwise covered
- Minimal Sum of Products (SOP)
  - Contains sum of prime implicants

Quine's Theorem with Don't Cares

- Similar to Standard Tabular Method
  – f(x,y,z) = yz'+xy'z
- Also specify Don't Care Terms
  – d(x,y,z)=x'z
- Calculate minterm canonical form
  - Place canonical minterms in left column
    – Same as before
  - Identify don't cares with 'd'
  - Group by number of complemented literals
    – Preserve d iff all terms are don't care.
  - Compare between successive group
    – Same as before
- Consider: f(w,x,y,z) = ∑(2,5,6) + d(1,3)

Distance-1 Merging

- SOP formula is a **Complete Sum** iff
  1. No term includes other term
  2. Consensus of any two terms:
     – Does not exist -or-
     – Is contained in some term of the formula
- Consensus
  – Xy + Xy' = X

Iterated Consensus

- Starting with a SOP formula
  - For each pair of elements
    - If terms match for all but 1 complemented term
    - Join terms and add consensus
  - Eliminate inclusive terms.
- Consider: f(w,x,y,z) = wx + x'y + xyz

Recursive Computation of Prime Implicants

- Starting with 2 complete sums F_1 & F_2, a complete sum $F_1 \bullet F_2$ is obtained if
  - Multiply out F_1 & F_2 using the idempotent and distributive properties and x •x'=0
  - Eliminate all terms that are contained in some other forms
- Consider: f(w,x,y,z) = (w+x) (x'+y) (y+z)

Recursive Computation of Prime Implicants

- If a SOP formula is given, use the second form of Boole's Expansion Theorem
  - $f(x_1,x_2,\ldots,x_n)=[x_1'+f(1,x_2,\ldots,x_n)]\bullet[x_1+f(0,x_2,\ldots,x_n)]$
  - Apply the same procedure to each of $f(1,x_2,\ldots,x_n)$ & $f(0,x_2,\ldots,x_n)$ until they simplify
    - The simplification is guaranteed to take place
  - The complete sum can then be reconstructed by $[x1'+CS(f(1,x_2,\ldots,x_n))]\bullet[x_1+CS(f(0,x_2,\ldots,x_n))]$
    - Remove absorbed terms if necessary

Unate Covering Problem (UCP)

- The problem of finding a minimal expression for a given function $f(x_1,x_2,\ldots,x_n)$
  - Prime Implicant Chart
  - Boolean Matrix
    - One row for each minterm
    - One column for each prime implicant
  - Essential Prime Implicant
  - Row Dominance
  - Column Dominance

Example of Essential Prime Implicants

– Minterms : { xyz , xyz' , x'yz}
– Prime Implicants : {xy , yz }

Prime Implicants

Minterms	P_1=xy	P_2=yz
M1=xyz	1	1
M2=xyz'	1	
M3=x'yz		1

– Both P_1 & P_2 are **Essential** to <u>cover</u> M_1, M_2 & M_3.

Essential Prime Implicant

- Essential Prime Implicant
  – A prime implicant that is not included in other prime implicant
  – All E.P.I.'s are parts of the minimal solution
- Identifying an E.P.I in a boolean matrix
  – Singleton = Row covered by only one column
  – If a row in has singleton, then the P.I. in that row is a E.P.I.
  – All the E.P.I. rows and all the minterm columns that are covered by them are eliminated from the boolean matrix to obtain a simpler one
- Consider: $f(w,x,y,z) = \sum(0,1,2,8,9,10,13,14,15)$

Row Dominance

- If a row r_i covers all the minterms that are covered by another row r_j, then r_i dominates r_j
  - If r_i dominates r_j, then r_j can be eliminated from the boolean matrix
    - Direct consequence of the absorptive property
    - As long as the cost of the P.I. in r_i is not greater than the cost of the P.I. in r_j
- Consider: $f(v,w,x,y,z) = \sum(0,1,3,4,7,13,15,19,20,22,23,29,31)$

Column Dominance

- If a column c_i is covered by all the PI's in another c_j, then c_i dominates c_j
  - If c_i dominates c_j, then c_i can be eliminated from the boolean matrix
    - Direct consequence of the consensus property
    - As long as the cost of the PI in c_j is not greater than the cost of the P.I. in c_i
- Consider: $f(v,w,x,y,z) = \sum(0,1,3,4,7,13,15,19,20,22,23,29,31)$

Multiple Output Functions

- In order to find minimal expression, we need to find all the multiple output prime implicants
  - If a multiple output function is composed of 3 single output functions f_1, f_2, f_3, then we need to consider
    - The prime implicants of f_1, f_2, f_3
    - The prime implicants of $f_1 \bullet f_2$, $f_1 \bullet f_3$, $f_2 \bullet f_3$
    - The prime implicants of $f_1 \bullet f_2 \bullet f_3$
- Use a combined boolean matrix to solve
- Consider: $f_1(w,x,y,z) = \sum(3\text{-}7,11\text{-}14)$ &
  $f_2(w,x,y,z) = \sum(3,5,11,13,15)$