CSE460
Switching Theory

Lecture 3 : Partial Orders, Hasse Diagrams, Meet, Join, Poset, Ordered Sets, Lattices

Washington University
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http://www.arl.wustl.edu/~lockwood/class/coe460/

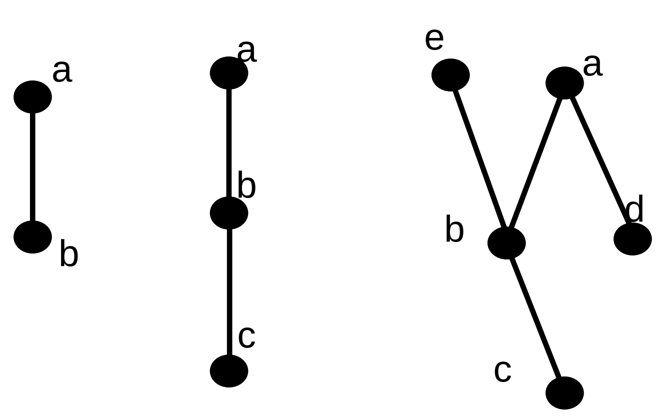
Announcements

- Reading
  - Hachtel: Chapter 3 - Boolean Algebras (continued)

Review of Partial Orders

- Partial Orders: Relations that are:
  - Reflexive
    - For every $x \in A$, $x\Re x$
      - $(x,x) \in \Re$, Relation exists in all elements of diagonal
    - Graph nodes have an implicit pointer to self
  - Antisymmetric
    - If both $(x,y)\in\Re$ and $(y,x)\in\Re$ then $x=y$
      - (x,y) but no (y,x) for all $x\neq y$
    - Graph contains only unidirectional links
  - Transitive
    - For every x,y,z, if $(x,y)\in\Re$ and $(y,z)\in\Re$ then $(x,z)\in\Re$
    - Graph need not include transitive edges

Hasse Diagrams



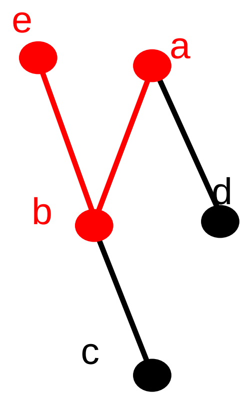
↑assumed

a>b ; {(a,a),(b,a),(b,b)}

• Meet

- Greatest Lower Bound
- x•y : **Meet** of x and y

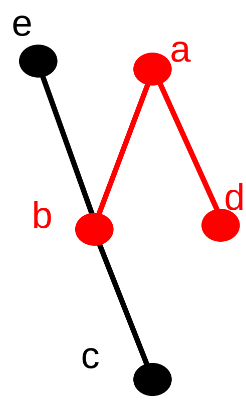
- b = a•e



+ Join

- Least upper bound
- x+y : **Join** of x and y

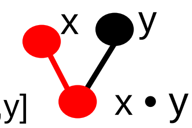
- a=b+d



Theorems for a Poset

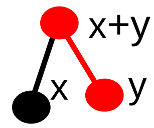
- If x and y have a greatest lower bound

  $x \geq (x \bullet y)$ [x is at least as large as the meet of x,y]

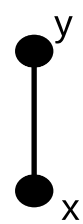


- If x and y have a least upper bound

  $y \leq (x+y)$ [y is at least as small as the join of x,y]



- Dual :
  - Replace $\geq$ with $\leq$
  - Replace • with +



- Therefore

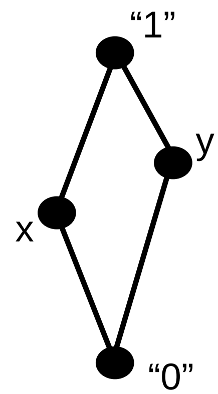
  $x \leq y \Leftrightarrow x \bullet y = x$ *and* $x + y = y$

Totally Ordered & Well-ordered sets

- **Totally ordered**
  - All pairs are comparable
  - Example: Real numbers
- **Well-ordered**
  - Subset of totally ordered set
  - Has an smallest element
- **Principle of Induction**
  - Given that P(0) is true
  - Given that for all $(n \in N) > 0$,
    that if P(n-1) is true then P(n) is true
  - Then, P(n) is true.

Lattice

- Poset where any two elements have
  - Meet (•)
  - Join (+)
- Greatest Element = 1
- Least Element = 0



Properties of a Lattice

- Idempotent
  - x+x=x
  - x•x=x
- Commutative
  - x+y=y+x
  - x•y=y•x
- Associative
  - x+(y+z)=(x+y)+z
  - x•(y•z)=(x•y)•z
- Absorptive
  - x•(x+y)=x
  - x+(x•y)=x