CSE 460
Switching Theory
Lecture 6 :

Implicants, Prime Implicants, Essential Prime Implicants, Unate Covering Problem, Quine's Theorem,

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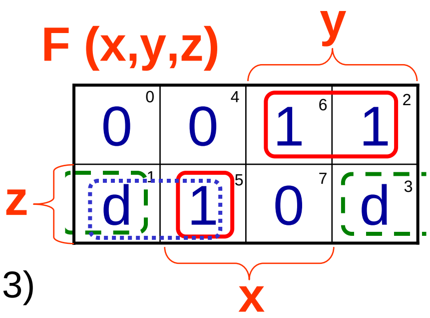
Implicants

- Implicant
  - – Product term included in f
  - – Example: {xy , yz , xyz , xyz' , x'yz} in f=xy+yz
- Prime Implicant
  - – Implicant not included in other implicant
  - – Example: { xy , yz },
    - But not { xyz } because {xyz} ∈ { yz }
- Essential Prime Implicant
  - – Implicant includes term not otherwise covered
- Minimal Sum of Products (SOP)
  - – Contains sum of prime implicants

Graphical view of On-Set & Don't care terms

- On set
  - $f(x,y,z) = \boxed{yz'} + \boxed{xy'z}$
- Don't care terms
  - $d(x,y,z) = \boxed{x'z}$



0	0	1	1
d	1	0	d

- Alternate Expression
  - $f(x,y,z) = \sum(2,5,6) + d(1,3)$
- Implicants = xyz', xyz', xy'z'
- Minimal Resulting Function with don't cares
  - $f(x,y,z) = \boxed{yz'} + \boxed{y'z}$

Quine's Theorem with Don't Cares

- Calculate minterm canonical form
  - Place canonical minterms in left column
  - Identify don't cares ('d') with a dash ('-')
  - Group by number of complemented literals
    - Preserve d iff all terms are don't care.
  - Compare between successive group

Distance-1 Merging

- SOP formula is a **Complete Sum** iff
  1. No term includes other term
  2. Consensus of any two terms:
     - Does not exist -or-
     - Is contained in some term of the formula
- Consensus
  - Xy + Xy’ = X

Iterated Consensus

- Starting with a SOP formula
  - For each pair of elements
    - If terms match for all but 1 complemented term
    - Join terms and add consensus
  - Eliminate inclusive terms.
- Consider: f(w,x,y,z) = wx + x’y + xyz

Unate Covering Problem (UCP)

- The problem of finding a minimal expression for a given function f(x_1,x_2,…,x_n)
  - – Prime Implicant Chart
  - – Boolean Matrix
    - One row for each minterm
    - One column for each prime implicant
  - – Essential Prime Implicant
  - – Row Dominance
  - – Column Dominance

Example of Essential Prime Implicants

- – Minterms : { xyz , xyz' , x'yz}
- – Prime Implicants : {xy , yz }

		Prime Implicants	
		P_1=xy	P_2 =yz
Minterms	M1=xyz	1	1
	M2=xyz'	1	
	M3=x'yz		1

– Both P_1 & P_2 are **Essential** to cover M_1, M_2 & M_3.

Essential Prime Implicant

- Essential Prime Implicant
  - A prime implicant that is not included in other prime implicant
  - All E.P.I.'s are parts of the minimal solution
- Identifying an E.P.I in a boolean matrix
  - Singleton = Row covered by only one column
  - If a row in has singleton, then the P.I. in that row is a E.P.I.
  - All the E.P.I. rows and all the minterm columns that are covered by them are eliminated from the boolean matrix to obtain a simpler one
- Consider: $f(w,x,y,z) = \sum(0,1,2,8,9,10,13,14,15)$

Row Dominance

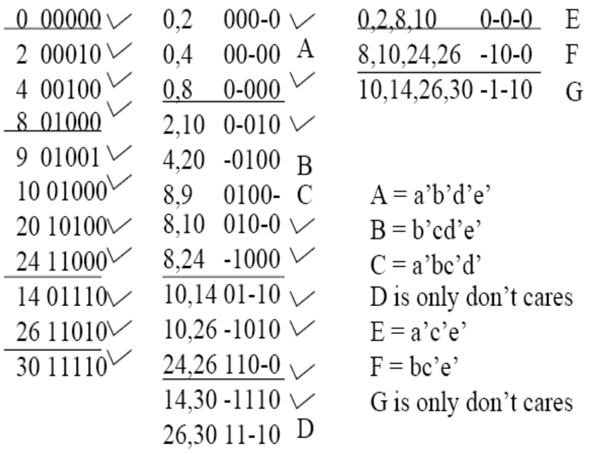
- If a row r_i covers all the minterms that are covered by another row r_j, then r_i dominates r_j
  - If r_i dominates r_j, then r_j can be eliminated from the boolean matrix
    - Direct consequence of the absorptive property
    - As long as the cost of the P.I. in r_i is not greater than the cost of the P.I. in r_j
- Consider: $f(v,w,x,y,z) = \sum(0,1,3,4,7,13,15,19,20,22,23,29,31)$

Column Dominance

- If a column c_i is covered by all the PI's in another c_j, then c_i dominates c_j
  - – If c_i dominates c_j, then c_i can be eliminated from the boolean matrix
    - Direct consequence of the consensus property
    - As long as the cost of the PI in c_j is not greater than the cost of the P.I. in c_i
- Consider: f(v,w,x,y,z) = $\sum$(0,1,3,4,7,13,15,19,20,22,23,29,31)

In-Class Example with 5 Variables

$$f_B(a,b,c,d,e) = \sum(0,2,8,9,20,24) + \sum_d(4,10,14,26,30)$$



0	00000	✓	0,2	000-0	✓	0,2,8,10	0-0-0	E
2	00010	✓	0,4	00-00	A	8,10,24,26	-10-0	F
4	00100	✓	0,8	0-000	✓	10,14,26,30	-1-10	G
8	01000	✓	2,10	0-010	✓			
9	01001	✓	4,20	-0100	B			
10	01000	✓	8,9	0100-	C			
20	10100	✓	8,10	010-0	✓			
24	11000	✓	8,24	-1000	✓			
14	01110	✓	10,14	01-10	✓			
26	11010	✓	10,26	-1010	✓			
30	11110	✓	24,26	110-0	✓			
			14,30	-1110	✓			
			26,30	11-10	D			

A = a'b'd'e'
B = b'cd'e'
C = a'bc'd'
D is only don't cares
E = a'c'e'
F = bc'e'
G is only don't cares

5-Variable Example (continued)

- Essential Prime Implicants

		✓	✓	✓	✓	✓	✓
		0	2	8	9	20	24
	A	x					
✓	B					ⓧ	
✓	C	x		x	ⓧ		
✓	E	x	ⓧ	x			
✓	F			x			ⓧ

- Final Solution

$$f(a,b,c,d,e) = b'cd'e' + a'bc'd' + a'c'e' + bc'e'$$

Multiple Output Functions

- In order to find minimal expression, we need to find all the multiple output prime implicants
  - If a multiple output function is composed of 3 single output functions f_1, f_2, f_3, then we need to consider
    - The prime implicants of f_1, f_2, f_3
    - The prime implicants of $f_1 \bullet f_2$, $f_1 \bullet f_3$, $f_2 \bullet f_3$
    - The prime implicants of $f_1 \bullet f_2 \bullet f_3$
- Use a combined boolean matrix to solve
- Consider: $f_1(w,x,y,z) = \sum(3\text{-}7,11\text{-}14)$ &
  $f_2(w,x,y,z) = \sum(3,5,11,13,15)$