1. (6 points). One of our basic lemmas for the partition data structure states that the number of nodes in tree with root $x$ is at least $2^{\text{rank}(x)}$. Consider the following more general property: for any node $x$, the number of nodes in the subtree rooted at $x$ is at least $2^{\text{rank}(x)}$. Explain why this is true if we do not use path compression, and why it is not true if we do use path compression. Hint. Since the condition is true for all tree roots, it is true at node $x$ just before $x$ becomes a non-root.
2. (4 points) If $a$ is a node in a partition data structure, with $\text{rank}(a) = 5$ and $\text{level}(a) = 2$, what is the smallest possible value for $\text{rank}(p(a))$? If $a$ is non-singular, what is the smallest possible value of $\text{rank}(x)$ where $x$ is the root of the tree containing $a$?