1. (15 points). Consider the network shown below

Assume that the network implements ABR flow control and that there are ABR virtual circuits joining A and A’, B and B’, C and C’, D and D’. Assume all links are operating at 150 Mb/s, that the output buffer at switch X on the link to Y has sufficient capacity to hold 450 Kbits and the delays experienced by cells are as indicated indicated by the labels in the figure. Also, assume that at time 0, A and D are sending data at 100 Mb/s and 25 Mb/s respectively. Now suppose that after 10 ms, B starts sending a burst at 140 Mb/s and C starts sending at 35 Mb/s. At what time does the buffer become full?

Data from B and C arrives at X starting at 12 ms. At this point, the rate at which data is entering the buffer is 100+25+140+35=300 Mb/s, so buffer fills at rate of 150 Mb/s or 150 Kb/ms. So buffer fills at time 12+450/150=15 ms.

Assuming the ABR controller attempts to allocate the bandwidth fairly among the competing sources and that 90% of the link bandwidth is allocated during an overload period, how much bandwidth will each of the four sources be assigned by the controller, assuming that none is given more than it wants? (A wants 100, B wants 140, C wants 35 and D wants 25.)

135 Mb/s is to be allocated. One fourth of this is about 34 Mb/s, but since D can only use 25 Mb/s, part of its share can be distributed among the others. If we give D its 25 Mb/s, that leaves 110 Mb/s to be distributed among the other three, which is about 37 Mb/s each. But since C can only use 35 Mb/s, some of its share can be given to A and B. So giving C 35 Mb/s and D 25 Mb/s, leaves 75 Mb/s to split between A and B. So A and B each get 37.5 Mb/s.
At what time do A, B, C and D change their rates.

A changes at 12+5=17 ms and B changes at 12+8=20 ms. C and D do not change their rates.

At what time does the rate into the buffer drop below the link rate?

Both A and B must reduce their rates before the rate entering the buffer drops below the link rate. Also, once A and B change their rates, there is still a delay of 1 ms for A and 2 ms for B, before the effect of the rate changes are felt at the buffer. So, the rate into the buffer drops below the link rate at time 22 ms.

Approximately how much data is lost during the period of time when the buffer is full?

From time 15 to 18, the rate into the buffer is 300 Mb/s. So 3×150 Kb=450 Kb is lost during this time period. From 18 to 22, the rate into the buffer is 237.5 Mb/s, so 4×87.5=350 Kb is lost during this time period. Total lost is thus 800 Kb.

2. (10 points) Consider an eight port crossbar-based switch, in which each IPP has a single queue and the crossbar controller uses a time-slotted arbitration ring to select which inputs send cells to each output in a given cycle. Suppose the input buffers contain cells for various outputs, as indicated below.

*input 0: ②, 4, X means cells for outputs 2, 4, 1 with cell for output 2 first*
*input 1: ③, ②, X*
*input 2: ②*
*input 3: ⑥, ④*
*input 4: ④, 3, 7, 3*
*input 5: X, 4, 2*
*input 6: ⑤, X, 7*
*input 7: ④*

Assume that the time slotted arbitration ring rotates the starting point of the arbitration at the start of each cell cycle. On the first cell cycle, input i gets the first chance at output i, on the second cell cycle, input i gets the first chance at output i + 1, etc.

After the first cell cycle, which cells have been sent to the output? Show this, by circling the selected cells in the lists above. Which cells are selected in the second cycle? (Underline these ones) Which cells are selected in the third cycle (make an ‘x’ through these.)

3. (10 points) Consider the simple discrete time queuing model in which cells arrive independently each cycle with probability λ, leave with probability µ and an arriving cell is discarded if the queue is full and no cell leaves in that time step. Give equations for the steady-state probabilities π(0),..., π(B) in terms of λ, µ and B.

\[
\begin{align*}
\pi(0) &= \frac{1 - \mu}{1 - \rho\mu} - \mu \\
\pi(j) &= (\rho^j/(1 - \mu))\pi(0) \quad \text{where } \rho = \lambda(1 - \mu)/\mu(1 - \lambda)
\end{align*}
\]

Give expressions for the average queue length and for the cell loss probability.

\[
\text{average queue length} = \sum_{i=1}^{B} i\pi(i)
\]

\[
\text{cell loss probability} = (1 - \mu)\pi(B)
\]
4. (10 points) Give expressions for the mean and variance of the binomial distribution \( B(n, p) \), the geometric distribution \( G(p) \) and for the exponential distribution with parameter \( \alpha \).

   For \( B(n, p) \), \( \mu = np \) and \( \sigma^2 = np(1 - p) \).

   For \( G(p) \), \( \mu = 1/p \) and \( \sigma^2 = (1 - p)/p^2 \).

   For exponential distribution with parameter \( \alpha \), \( \mu = 1/\alpha \) and \( \sigma^2 = 1/\alpha^2 \).

5. (10 points) Consider an ATM switching system with 64 ports and a crossbar interconnection network. If the input and output links each support 2048 virtual circuits and if at most 25\% of the outgoing VCIIs are used by multicast virtual circuits, how many routing table entries are needed in each input port processor? How many bytes per entry, assuming both unicast and multicast virtual circuits. You may assume that the system only switches on VCIIs (VPIs are ignored on input and set to 0 on output).

   Number of entries is 2048. The number of bytes per entry is

   \[
   (1/8) \max \left\{ 1 + 6 + 16, 1 + \left[ \log_2(1/4) \cdot 2048^64 \right] + 64 \right\} = (1/8)79 = 10 \text{ bytes}
   \]

   How many entries are needed in each OPP routing table? How many bytes per entry?

   \( 2^{14} = 16,384 \text{ entries per OPP table. 2 bytes each.} \)

6. (10 points) Suppose you are designing a bus-based switch with 32 ports and 600 Mb/s links. Assume your circuit technology allows you a clock rate of \((512/n) \text{ MHz}\) where \( n \) is the number of loads on each bus line. Assuming a conventional, single-level bus, how many bus lines are needed to provide the bandwidth to accommodate 100\% loading on all links? (You may neglect added bandwidth needed for internal control information.)

   The clock rate is \( 512/(2 \times 32) = 8 \text{ MHz, so the required bus width is 32 \times 600/8 = 2400.} \)

   How many pins are required to connect each IPP and OPP to the bus?

   4,800 per IPP/OPP pair.

   Suppose you change the design so to use a ring, instead of a bus. How many pins are required per IPP and OPP in this case?

   The clock rate is \( 512/2 = 256 \text{ MHz, so the ring width is 32 \times 600/256 = 75 and the number of pins per IPP/OPP pair is 150.} \)

   Suppose you use a crossbar instead of a ring, and that the crossbar chips can have at most 64 input pins and output pins per chip. How many crossbar chips do you need, assuming the total crossbar bandwidth must be at least twice that of the external links? How many pins are needed per port (counting IPP, OPP and crossbar pins). You may neglect the crossbar controller.

   Since there are 64 pins and 32 ports, we can implement the crossbar using several chips in parallel and no tiling. So we can have a clock rate of 256 MHz. The number of chips needed is \( [2 \times 600/256] = 5 \) and the number of pins per IPP/OPP pair is 20.

7. (10 points) Consider a four port bus-based switch in which the traffic distribution from inputs to outputs is distributed randomly, but unevenly as follows.

   Input 0 sends half of its arriving traffic to output 1, one-third to output 2 and the rest to output 3.
Input 1 sends one-fourth of its arriving traffic to output 0, one-third to output 2 and the rest to output 3.

Input 2 sends one-third of its arriving traffic to output 0, one-fourth to output 1 and the rest to output 3.

Input 3 sends one-half of its arriving traffic to output 0, one-third to output 1 and the rest to output 2.

What is the probability that in any given cell cycle, output 0 receives no cells, one cell, two cells, three cells or four cells? Assume that all inputs receive cells on their input links with probability $p$ and that the outputs for all cells are independent.

\[
\begin{align*}
\text{Pr}\{0 \text{ cells}\} & = (1 - p/4)(1 - p/3)(1 - p/2) \\
& = (1/24)(4 - p)(3 - p)(2 - p) = (1/24)(24 - 26p + 9p^2 - p^3) \\
& = (p/24)((3 - p)(2 - p) + (4 - p)(2 - p) + (4 - p)(3 - p)) = (p/24)(26 - 18p + 3p^2) \\
\text{Pr}\{2 \text{ cells}\} & = (1 - p/4)(p/3)(p/2) + (p/4)(1 - p/3)(p/2) + (p/4)(p/3)(1 - p/2) \\
& = (p^2/24)((4 - p) + (3 - p) + (2 - p)) = (p^2/24)(9 - 3p) \\
\text{Pr}\{3 \text{ cells}\} & = (p/4)(p/3)(p/2) = p^3/24 \\
\text{Pr}\{4 \text{ cells}\} & = 0
\end{align*}
\]

What is the average number of cells received by output 0 each cell cycle?

\[
((1/2) + (1/3) + (1/4))p = (6 + 4 + 3)/12 = (13/12)p
\]

8. (15 points) In the diagram shown below, all of the six terminals are participating in a multi-party video conference. With current ATM standards, this typically requires that each terminal have a one-to-many virtual circuit to all of the others. The network reserves bandwidth for each of these virtual circuits independently. Assuming, each one-to-many video channel has a bandwidth of 15 Mb/s and is set up as a CBR virtual circuit, how much bandwidth will the network allocate on each link in each direction? (Show your answer on the diagram.)
If there are never more than two cameras active, what is the maximum bandwidth that can actually be used on each link? (Show usage in both directions.)

Suppose we want to use VBR virtual circuits, instead of CBR, to take advantage of the fact that each user is only actively transmitting part of the time. Assuming that each camera is turned on for about the same amount of time during the conference, what would you specify as the average rate for each of the one-to-many VCs (assume that at all times, exactly two out of the six cameras are turned on, but which pair is on changes, as different people talk).

\[ 15 \times (2/6) = 5 \text{ Mb/s} \]

For a VBR virtual circuit, we also need to specify a token reservoir size to bound the length of a burst. For the conference application, the size of the token reservoir determines how long one camera can remain active before switching off. How large should the token bucket be if we want to allow for one speaker’s camera to stay on for five minutes without interruption? (Assume that the rate at which tokens enter the token reservoir is the average rate you gave in the previous part of the question.)

If one speaker sends at 15 Mb/s, he consumes \((15 \text{ Mb})/(424 \text{ bits}) \approx 35,000 \text{ tokens/second},\) so uses 10,500,000 tokens in 5 minutes (=300 seconds). Since the token reservoir is also filling at a rate of \(35,000/3 \text{ tokens per second},\) the reservoir only needs to hold \(10,500,000 - 300 \times 35,000/3 = 7,000,000.\)