Exam 2, Part A
Solutions

1. (12 points) Show how to calculate the perfect shuffle function \( \tau_{a,b}(x) \).

\[
\tau_{a,b}(x) = (x \mod a)b + \lfloor x/a \rfloor
\]

Define the series operator for network construction.

If \( N_1 \) is a network with \( n_1 \) outputs and \( N_2 \) is a network with \( n_2 \) inputs, then

\[
N_1 \times N_2 = (n_2 \cdot N_1) \circ \tau_{n_1,n_2}; (n_1 \cdot N_2)
\]

What is the associative property for the series operator?

\[
N_1 \times (N_2 \times N_3) \approx (N_1 \times N_2) \times N_3
\]

where \( \approx \) denotes strong isomorphism.

2. (15 points) Draw a picture of the network defined by the expression \( X_{2,3} \circ (X_{2,3} \times X_{3,3}) \circ X_{3,3} \).
How many paths are there between an input \(x\) of this network, and an output \(y\)?

3

Suppose this network is used in a dynamic routing mode for point-to-point traffic. What is the maximum possible value for \(\lambda_1(C)\) for a link \(\ell\) in stage 1 of this network, where \(C\) is any collection of virtual circuits that place a load of no more than \(\beta\) on any input or any output? (2/3)\(\beta\)

What if \(\ell\) is in stage 2? Stage 3? (4/3)\(\beta\) and \(\beta\)

3. (12 points) When building a switching system to support moderate speed links (say 150 Mb/s), one can use internal data paths having bandwidth comparable to the external links, or one can use internal data paths that are many times faster than the external links, with many lower speed links multiplexed onto a single high speed switch port. List four advantages of using a high speed core.

(a) Sharing of common circuitry over external links.
(b) Better queueing performance for bursty traffic.
(c) Can handle higher speed links as well as low speed links.
(d) Reduced complexity and delay in interconnection network.
(e) Less fragmentation of bandwidth and memory.

4. (12 points) Give the definition of \(D_{n,d}\).

For \(n = d^k\) where \(d\) is an integer,

\[
D_{n,d} = \begin{cases} 
X_{d,d} & \text{if } n = d \\
X_{d,d} \times X_{n/d,d} & \text{if } n > d 
\end{cases}
\]

Give the definition of \(Y_{n,d}\).

For \(n = d^k\) where \(d\) is an integer,

\[
Y_{n,d} = \begin{cases} 
X_{d,d} & \text{if } n = d \\
\tau_{n/d,d} \cdot X_{d,d} \times Y_{n/d,d} & \text{if } n > d 
\end{cases}
\]

Give the definition of \(\Omega_{n,d}\).

For \(n = d^k\) where \(d\) is an integer,

\[
\Omega_{n,d} = \Omega_{n,d}^k
\]

where

\[
\Omega_{n,d}^i = \begin{cases} 
\tau_{n/d,d} \cdot ((n/d) \cdot X_{d,d}) & \text{if } i = 1 \\
\Omega_{n,d}^1 \cdot \Omega_{n,d}^{i-1} & \text{if } i > 1 
\end{cases}
\]

Give the definition of \(B_{n,d}\).

For \(n = d^k\) where \(d\) is an integer,

\[
B_{n,d} = \begin{cases} 
X_{d,d} & \text{if } n = d \\
X_{d,d} \odot B_{n/d,d} \odot X_{d,d} & \text{if } n > d 
\end{cases}
\]
5. (10 points) Consider a multistage network in which each switch element has four inputs and four outputs, and there is an eight slot output queue associated with each output. Suppose the four queues contain 3, 7, 5 and 4 cells, respectively. If the system uses grant flow control, how many grants can be sent to the upstream neighbors, at the start of the cell cycle? Why?

*Just 1, since the second queue only has room for a single cell.*

Suppose the system uses acknowledgement flow control and each of the upstream neighbors sends a cell to the given switch element. What is the maximum number of cells that may be acknowledged by the switch element in this situation? Why?

*Four cells may be acknowledged, so long as no more than one of the arriving cells is addressed to the second output, and no more than three are addressed to the third output.*

What is the minimum number? Why?

1, *since if all arriving cells are addressed to the second output, only 1 can be stored in the queue.*

Suppose that the switch element uses a single shared buffer with a total capacity of 24 cells, and the queue contains 3 cells for output 0, 7 cells for output 1, 5 for output 2 and 4 for output 3. In this case, how many grants will be given to the upstream neighbors, if grant flow control is used? Why?

*In this case 4 grants can be given, since there are 19 occupied cell slots and 5 empty slots that can accept arriving cells.*

If acknowledgement flow control is used, how many acks will be given? Why?

4, *for the same reason as above.*

6. (12 points) Consider a switching system that uses binary copying and cell recycling to implement multicast (as in the WUGS switch). Suppose the system supports 64 external links of 600 Mb/s, has a maximum virtual circuit rate of 150 Mb/s and may have as much as 25% of its total capacity used by multicast virtual circuits. If all ports on the core switch are used to carry both traffic from external links and recycling traffic, what is the minimum bandwidth that the internal data paths must have to avoid blocking of multicast virtual circuits?

\[
\frac{1}{\beta} \geq 1 + \theta + \frac{B}{\beta} = 1 + 0.25 + 0.25 = 1.5, \text{ so the internal bandwidth must be at least } 1.5 \times 600 = 900 \text{ Mb/s.}
\]

Suppose that instead of having all ports shared by recycling traffic and external traffic, that some ports are dedicated to recycling, while others are dedicated to external traffic. If the internal data paths have the same bandwidth as the external links, how many recycling ports are required, assuming that you still have 64 external links?

\[
h \geq \frac{bn}{\theta + (1/\beta) - (B/\beta)} = \frac{0.25n}{0.25 + 1 - 0.25} = n/4 \text{ where } h \text{ is the number of recycling ports and } n \text{ is the total number of links. If } n - h = 64, \text{ then } 4h - h \geq 64 \text{ so } h \geq 22.
\]

Which of these two switching systems is the least expensive, if the cost of a switching system is proportional to the number of ports, and is proportional to the bandwidth of the internal data paths?

*Using these assumptions about cost, the ratio of the cost of the first system to the cost of the second system is (1.5)(64/86) = 1.12, so the first system is 12% more expensive than the second.*
7. (10 points) Consider a switching system that uses binary copying and cell recycling to implement multicast. How many virtual circuit table entries are used by a one-to-many multicast with 25 outputs?

24, counting the one at the input port and the ones at all the recycling ports.

Suppose we have a many-to-many multicast application with 26 participants, and we implement it using 26 separate one-to-many multicasts. How many table entries are needed in this case?

24 for every 1-to-25 multicast, so 26 × 24 = 624 table entries.

How many table entries must be changed to add another participant to this multicast application?

The new participant must be added to all 26 existing multicasts, at a cost of two table entry changes for each one, giving 52. In addition, a new 1-to-26 multicast must be created, which uses 25 entries. So the total is 77 entries.

Suppose we instead implement this application using a single shared many-to-many multicast tree and upstream discard. In this case, how many table entries are required for 26 participants?

26 for the initial entry to forward cells from input ports into the shared tree and 25 for the shared tree. So 51, altogether.

How many entries must be changed to add a new participant?

3

8. (8 points) Give an expression for the combinatorial power of $Y_{125,5}$.

$(5!)^{25\times3}/125!$

Given an expression for the combinatorial power of $D_{256,4,3}$.

Since this network is isomorphic to a Benes network, it has the same combinatorial power as the Benes network. The combinatorial power is 1.

9. (12 points) Consider a switching system which uses a static routing network with topology $D_{256,4,2}^*$. Assume that the external links have 4096 entries and up to one virtual circuit in eight may be associated with a multicast virtual circuit. Also assume that the network’s internal data paths are configured to be 3 times faster than the external links. How many multicast routing tables are required in the switching network, assuming there is one per switch element?

$6 \times 64 = 384$ tables.

How many entries does each routing table require if global multicast identifiers are used and the routing tables use direct lookup? How many bits per entry?

$(1/8)(4096)(256) \approx 130,000$ entries and 4 bits per entry.

How many entries if CAMs are used? Bits per entry?

$(1/8)(4)(4096)(3) = 6144$ entries and $4 + \log_2(1/8)(4096)(256) = 21$ bits per entry.

How many entries if local multicast identifiers are used, and are remapped at every table? How many bits per entry?
\[(1/8)(4)(4096)(3) = 6144 \text{ entries and } 4(1 + \log_2(1/8)(4096))(3) = 48 \text{ bits per entry.} \]

Which option do you think is the least expensive? The most expensive?

*If the cost per bit in a CAM exceeds the cost per bit of SRAM by a factor larger than \((48/21)\), then the third option is the least expensive. Otherwise, the second option is least expensive. The first option is the most expensive, unless the cost per bit of a CAM exceeds the cost per bit of SRAM by a factor greater than \((130/6)(4/21) \approx 4\). Otherwise the second choice is most expensive. I would expect the first option to be most expensive and the third option to be least expensive.*