Exam 2, Part B
November 21, 1997

In this part of the exam, you may use one page of notes and a calculator.

1. (50 points) Let $N^{|2|} = N \times N, N^{|3|} = N \times (N \times N))$ and so forth. So for example, $D_{n,d} = X_{{d,d}^{|k|}}$ if $k = \log_d n$. Now, let

$$H(n, d_1, d_2, h, r) = \begin{cases} 
X_{d_1,d_1}^{[k]} \otimes X_{d_1,d_1}^{[k-h]} \otimes (X_{d_1,d_1}^{[h-r]} \times X^{[r]}_{d_1,d_2}) & \text{if } r \leq h \\
X_{d_1,d_1}^{[k]} \otimes (X_{d_1,d_1}^{[k-r]} \times X_{d_1,d_2}^{[r-h]}) \otimes X_{d_1,d_2}^{[h]} & \text{if } r \geq h
\end{cases}$$

Note that if $d_1 = d_2$ that $H(n, d_1, d_2, h, r) \approx D^{*}_{n,d_1,h}$. Draw pictures of $H_{16,2,3,2,1}$ and $H_{16,2,3,2,3}$. 


Suppose this network is used to implement a multicast ATM switch with dynamic routing (the first $h$ stages distribute traffic, the remaining $k$ stages route and copy cells to the outputs). Let $\lambda_\ell(c_j)$ be the load that a virtual circuit $(x_j, Y_j, \omega_j)$ places on a link $\ell$ where $x_j$ is an input, $Y_j$ is a set of outputs and $\omega_j$ is a weight between zero and 1, representing the bandwidth of the virtual circuit’s data stream, relative to the rate of the internal links. If $\ell$ is in stage $i$ and is on some path from $x_j$ to one or more outputs in $Y_j$, what is the load that $c_j$ induces on $\ell$ if $i \leq h$?

What if $i \geq k$?

What if $h < i < k$?

How many inputs can reach link $\ell$ if $i \leq h$?
What if $i \geq k$?

What if $h < i < k$?

How many outputs can be reached from link $\ell$ if $i \leq h$?

What if $i \geq k$?
What if $h < i < k$?

If the total traffic on each input and each output is $\leq \beta$, what is the maximum load that can be placed on link $\ell$ if $i \leq h$?

What if $i \geq k$?

What if $h < i < k$?
What speed advantage is required to make this network nonblocking?
2. (50 points) The WUGS switch includes a range-copy mechanism that can be used to multicast a cell to a set of consecutive outputs in a single pass through its network. We can use this to implement an arbitrary multicast virtual circuit addressed to $2f$ different outputs, by first copying a cell to $f$ consecutive ports, then recycling the copied cells back to the inputs and doing a VXT lookup on each copy, to get a pair of output ports and VXIs. The resulting cells are then sent into the switching network again, using the standard binary copy mechanism. This allows any multicast to be done in just two passes through the network, rather than requiring $\log_2 f$ passes.

Suppose we want to set up a new multicast virtual circuit with fanout $2f$ and bandwidth $B$. How should we select a consecutive range of recycling ports to accommodate this new virtual circuit?

If the multicast traffic can be as much as 20% of the total outgoing traffic, what is the total traffic on all the recycling paths, assuming that there are $n$ outgoing links with a load of at most $\beta$ each and each port is shared by external traffic and recycling traffic.

What condition must be satisfied to guarantee that we can always add a new virtual circuit?
with fanout $2f$ and bandwidth $B$? (You may assume that $f$ divides evenly into $n$.)

What does this imply about the speed advantage that the system requires in order to be nonblocking?

If $(\beta/B) = 16$, what is the actual value of the required speed advantage for virtual circuits with fanout 4, 16 and 64? How does this compare with the case where we use binary copying and allow the number of recyling passes to increase with the fanout?
How would the results above change, if the load on different recycling paths were guaranteed to differ by at most some value $\Delta$?