1. Consider an output queue for a 150 Mb/s link in a bus-based switch, and assume that the data flowing through the queue originates from a set of WWW servers. Assume that each file transfer is 100 Kbytes and that one file is sent on each virtual circuit every 10 seconds (on average). To give the users good interactive response, we want it to take at most one second from the time the user clicks on an item until it is completely transferred to the user’s computer. Assume that the user’s computer contributes 50 ms of delay when sending the request, another 50 ms when receiving the response and that the server contributes 100 ms of delay in responding to the request. Also, assume that the one way propagation delay through the network is 50 ms. What rate, should each file be sent at to meet the objective of a one second response time?

The computers and the propagation delay contribute a total of 300 ms, leaving 700 ms available in which the data can be sent. Since the file is 100 Kbytes long, it must be sent at a rate of 800/.7 Kb/s or 1.14 Mb/s.

What is the average rate at which data is sent on each virtual circuit?

800 Kb every 10 seconds or 80 Kb/s.

How many virtual circuits of this type can share a link, if we want to limit the congestion probability to .01?

With a peak of 1.14 Mb/s and an average of 80 Kb/s, the peak-to-average ratio is 14.25:1, or \( p=.07 \). The number that can be active at the same time without exceeding the link rate is \( k=150/1.14=131 \). We want the largest \( n \) for which \( \sum_{i=k}^{n-1} \binom{n-1}{i} p^i (1-p)^{n-1-i} \leq .01 \). This is 1525.

What is the average link utilization in this case?

\[ .08 \times 1525/150 = .81. \]

What is the virtual cell loss under these conditions?

\[ \sum_{i=k}^{n-1} \binom{n-1}{i} p^i (1-p)^{n-1-i} ((i+1)\lambda - 1) / ((i+1)\lambda) = .00025 \]

How many virtual circuits can share the link with congestion probability .01 if each file is sent at 20 Mb/s instead of the rate you calculated?

Here we repeat the prior calculation with \( k=7 \) and \( p=.004 \) giving, \( n=585 \). The average link
utilization in this case is 0.31 and the virtual cell loss is .001.

How many if each file is sent at 75 Mb/s? What is the average link utilization and virtual cell loss in each of these cases?

Here we repeat the prior calculation with k=2 and p=.0011 giving, n=136, average link utilization of .072 and virtual cell loss of .0034.

Which choice of rates gives the best performance from the user’s standpoint? From the standpoint of the network service provider? Which, in your opinion, is the best choice overall?

In general, the user would prefer the smallest possible response time. With a peak rate of 75 Mb/s, the time to send the burst is about 10 ms, giving a response time of 310 ms instead of the 1 second obtained with a 1.14 Mb/s rate. The network provider would clearly prefer the smallest sending rate, since that allows the largest number of people to be served. The real difference between a 310 ms response time and a 1 second response time seems too small to make a real difference to the user’s subjective perception of the response time, so the original peak rate seems the best choice overall, since it allows the largest number of users to be served (more than 11 times the number that can be served with the 75 Mb/s sending rate).

2. (12 points) Derive the equations for the mean and variance of the geometric distribution, by applying the definitions given in the notes.

\[
E(x) = \sum_{i=1}^{\infty} i (1 - p)^{i-1} p = \frac{p}{1 - p} \frac{1 - p}{p^2} = \frac{1}{p}
\]

\[
E(x^2) = \sum_{i=1}^{\infty} i^2 (1 - p)^{i-1} p = \frac{p}{1 - p} \sum_{i=1}^{\infty} i^2 (1 - p)^i = \frac{p}{1 - p} \frac{(1 - p)(1 + (1 - p))}{p^3} = \frac{2 - p}{p^2}
\]

\[
\sigma^2 = E(x^2) - E(x)^2 = \frac{2 - p}{p^2} - \frac{1}{p^2} = \frac{1 - p}{p^2}
\]

Show that if \( x \) and \( y \) are geometric random variables with parameters \( p \) and \( q \) respectively, that \( z=\min\{x,y\} \) is also a geometric random variable, with parameter \( p+q-pq \).

Since \( x \) and \( y \) are geometric random variables, we can imagine two sequences of binary trials, one with success probability \( p \) and one with success probability \( q \). If we alternate trials of the two types, then \( z \) is simply the number of “rounds” until the first success of either type. Now, the probability of success in a single round is \( pq+p(1-q)+(1-p)q=p+q-pq \). Hence, \( z \) is a geometric random variable with parameter \( p+q-pq \).

3. Make a state diagram describing the behavior of the tail discard mechanism for maintaining packet integrity when \( r=4 \) and \( k=2 \), including the probabilities for the transitions among the states.
Write the balance equations for the state diagram and solve them to find the state probabilities for each state.

*The balance equations are given below*

\[
\begin{align*}
\pi(1100) &= \pi(0110)/3 \\
\pi(1001) &= \pi(1100) + \pi(0101)/3 + \pi(0110)/3 \\
\pi(0011) &= \pi(1001) + \pi(0011)/3 + \pi(0101)/3 \\
\pi(0110) &= \pi(0011)/3 \\
\pi(0101) &= \pi(0110)/3 + \pi(0011)/3 + \pi(1010) \\
\pi(1010) &= \pi(0101)/3
\end{align*}
\]

*Solving these we find*

\[
\begin{align*}
\pi(1100) &= 1/25, \pi(1001) = 4/25, \pi(0011) = 9/25, \\
\pi(0110) = 3/25, \pi(0101) = 6/25, \pi(1010) = 2/25
\end{align*}
\]

For this case, what is the goodput?

*The offered load is 2 and the states with a first bit of 1 have a total probability of 7/25, so the goodput is 14/25=0.56.*

What fraction of packets are lost?

\[.56/2=.27\] is the fraction of packets sent that are not lost, so the fraction lost is 0.73.

How does this compare with the fraction of cells that are lost?

*The fraction of cells that are lost is 0.5.*

4. Consider a 150 Mb/s link that is experiencing an extended overload period caused by 30 virtual circuits, each sending 4 Kbyte packets continuously at 50 Mb/s. Approximately what fraction of the cells sent on this virtual circuit are lost due to overflow of the queue at the sending end of the link?

*Since ten times as much traffic is being sent as the link can carry, 90% of the cells will be
lost due to queue overflow.

Assuming the queue controller does not implement early packet discard, or any similar method of preserving packet integrity, approximately what fraction of the packets sent are corrupted (that is, have at least one of their cells lost)?

Each packet requires about 85 cells. If 90% of the cells are lost, it’s very hard for any packet to escape without any of its cells being discarded. Therefore, the packet loss rate will be close to 100%.

Suppose the queue controller does implement early packet discard and that the buffer is large enough to ensure that in never overflows and never becomes empty during the overload period. What fraction of the packets are lost in this case?

90%

Approximately what fraction of the time is the buffer below the threshold?

Since 10% of the packets are getting through, and since these packets are starting when the buffer is below threshold, the time below threshold must be 10%.

Suppose we were to add another virtual circuit sending 4 Kbyte packets at 1 Mb/s. Approximately what fraction of packets sent by this new virtual circuit will be lost?

The rate for this VC is small enough that the buffer will still be below threshold about 10% of the time, so 90% of the packets from the new VC will be lost.

5. Consider an output queue in an ATM switch that uses Early Packet Discard (without hysteresis), to maintain packet integrity. Based on the worst-case analysis, how large a buffer is needed if we want to maintain 100% goodput for offered loads up to three times the link rate. Assume that the virtual circuit peak rate is 10 Mb/s, that the link rate is 150 Mb/s and the packet length is 4000 bytes.

\[ k=15, r=45 \text{ and } \lambda=1/15, \text{ so we need } (45-15)\ell =120,000 \text{ bytes above the threshold and } 15\ell =60,000 \text{ bytes below the threshold.} \]

Repeat the analysis for a link rate of 600 Mb/s and for a link rate 2.4 Gb/s.

For a link rate of 600 Mb/s, \( k=60, r=180 \text{ and } \lambda=1/60, \text{ so we need } (180-60)\ell =480,000 \text{ bytes above the threshold and } 60\ell =240,000 \text{ bytes below the threshold.} \)

For a link rate of 2.4 Gb/s, \( k=240, r=720 \text{ and } \lambda=1/240, \text{ so we need } (720-240)\ell =1,920,000 \text{ bytes above the threshold and } 240\ell =960,000 \text{ bytes below the threshold.} \)

Answer all the questions above again, using the even-offset analysis.
Since $r^2=3$, the excursion above threshold is $(1/2)(2r-3k)k(\ell/r)$ and the excursion below threshold is $(1/2)k^2(\ell/r)$. This gives 30,000 above threshold and 10,000 below for a 150 Mb/s link, 120,000 above and 40,000 below for a 600 Mb/s link and 480,000 above and 160,000 below for a 2.4 Gb/s link.

What buffer size is needed if the buffer controller uses EPD with hysteresis?

4,000 bytes above threshold and 4,000 bytes below are sufficient in all cases.

6. Consider a virtual circuit that is being monitored by a Usage Parameter Control mechanism with two reservoirs, one for limiting the peak rate, the other for limiting the average rate. Each time a cell is sent on the virtual circuit, one token is consumed from each of the two token reservoirs. If either of them does not have a token available, the cell is discarded. Assume that the peak rate reservoir can hold one token and that a new token is generated every 15 microseconds. Assume that the average rate reservoir has a capacity of 100,000 tokens and a new token is generated every 300 microseconds. If both reservoirs are full initially, for how long can the user send data at a rate of 15 Mb/s before the average rate reservoir is empty?

If the user sends data at 15 Mb/s (one cell every 28.3 $\mu$s) there is always a token available in the peak rate reservoir. Every 300 $\mu$s, 1 new token goes into the average rate reservoir, but an average of 10.6 tokens are consumed, meaning that the number of tokens is reduced by 9.6 tokens every 300 $\mu$s. If it has 100,000 tokens initially, it takes about $0.0003*100,000/9.6=3.1$ seconds for the average rate token pool to empty.

What is the most bursty repeated pattern of data transmissions that could be sent over this virtual circuit without causing any cells to be lost?

The most bursty traffic pattern is an on-off pattern in which the user sends a longest possible burst at the peak rate and then becomes idle before sending another maximum length burst. For this UPC mechanism, the maximum allowed rate is one cell every 15 $\mu$s or 28.3 Mb/s. By the same kind of analysis we did above, the user can send at this rate for $0.0003*100,000/19=1.58$ seconds before the token reservoir becomes empty. If the user then waits another $0.0003*100,000=30$ seconds, the average rate token reservoir will become full again and the pattern can repeat. So, the most bursty data pattern consists of 44.7 Mbit bursts, one every 31.58 seconds and each burst sent at a rate of 28.3 Mb/s.

7. Consider the crossbar-based switch shown below. In this switch, each input port processor maintains a separate queue for each of the output ports. In the figure, the numbers in the input queues represent the number of cells waiting to be sent to that output port. The number next to the queue represents the output port that the cells are addressed to. Draw a picture of the graph that represents the set of requests from inputs to outputs. Find a maximum weighted matching in this graph, where the weights are the numbers of waiting cells. Show the state of the input queues after the cells selected on the basis of this matching have been sent (that is, specify for each input queue, the number of waiting cells after the cells selected for transfer have been sent to the outputs). Repeat this for two more
iterations.
State of Switch after Second Cycle

Graph and Matching
State of Switch after Third Cycle