1. (15 points) Consider a 32 port version of the system described on page 3-65 of the lecture notes. Suppose that the topology of the interconnection network used with the system is $D_{32,2,2}^*$ and that the links operate at 600 Mb/s. If at most 10% of the outgoing traffic is multicast, what is the maximum amount of traffic that can be carried on any of the “recycling paths”. What is the maximum amount of traffic that can be present on a link in stage 1? What is the maximum amount of traffic on a link in stage 2? stage 3? stage 4? stage 5? stage 6? stage 7?.

2. (10 points) Consider the network $K_{n,d,h}$ defined by

$$K_{n,d,0} = D_{n,d}, \quad K_{n,d,h} = X_{d,d} \otimes K_{n/d,d,h-1} \otimes X_{d,d} \quad \text{for } h > 0$$

Show that $K_{n,d,h} \approx D_{n,d,h}^*$ by induction, using the associativity of the parallel connection operation.

3. (15 points) The digit reversal permutation $R_{k,d}$ is the permutation obtained by replacing each number in the range from 0 to $d^{k-1}$ with the number obtained by reversing the $k$ digits in its base $d$ representation. So, for example, $R_{4,2}$ maps the number 3 (binary representation 0011) to the number 12 (binary representation 1100). Show that the banyan network $Y_{27,3}$ is strongly isomorphic to the network obtained by preceding the delta network $D_{27,3}$ with the permutation $R_{4,2}$. Do this by drawing pictures of the two networks and labeling them so as to demonstrate their relationship. Prove by induction that this property holds in general. That is, that $Y_{n,d} \approx R_{k,d}; D_{n,d}$ where $k = \log_d n$.

4. (10 points) Find a set of routes that realizes the following permutation in the Benes network $B_{16,2}$. Use the graph coloring approach to determine how to partition the paths among the subnetworks at each step. For each step in the routing process, show the connection graph, the coloring you found and how the connections were divided among the subnetworks.

<table>
<thead>
<tr>
<th>input</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>output</td>
<td>14</td>
<td>4</td>
<td>11</td>
<td>15</td>
<td>2</td>
<td>13</td>
<td>0</td>
<td>3</td>
<td>7</td>
<td>1</td>
<td>6</td>
<td>12</td>
<td>5</td>
<td>10</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

5. (15 points) Suppose you are asked to design a switching system that uses static routing, has 2048 inputs and outputs, uses the three stage Clos network topology and supports external links of 1 Gb/s with virtual circuit rates from 0 up to 100 Mb/s. Assuming that the first stage switches have 64 inputs and the last stage switches have 64 outputs, how fast should the internal links be to make the system nonblocking for unicast traffic if there are 32 middle stage switches? How fast should they be to make it nonblocking for multicast traffic? How
fast to be reroutably nonblocking for multicast traffic? In this last case what is the fanout restriction in the first stage? Repeat the above, assuming 1 and 8 middle stage switches. Organize your answers in the form of a table with three rows and four columns.

6. (15 points) Consider an ATM switch using the topology $C_{4096,16,20}^5$. Assume that the external links have a bandwidth of 1 Gb/s and the internal links have a bandwidth of 1.5 Gb/s. Use Lee’s method to derive an expression for the blocking probability for this network when all virtual circuits have a bandwidth of 1 Gb/s. Express the blocking probability in terms of $p$, the expected fraction of each external link that is in use. What if every virtual circuit has a bandwidth of 500 Mb/s? What if every virtual circuit has a bandwidth of 300 Mb/s? Plot the blocking probability as a function of $p$ for values of $p$ between .5 and 1 for all three cases. Use a logarithmic scale for the y axis.

7. (10 points) Consider a contend-and-repeat network with the topology $X_{32,40} \otimes D_{256,4} \otimes X_{40,48}$. Explain how to compute the load on the output links, given the load on the input links, assuming uniform random traffic and assuming that the first stage performs random traffic distribution. Using your analysis, determine the maximum average traffic at the outputs. Let the bandwidth of the input links be 1 Gb/s. How does this maximum average output traffic under the idealized assumption that there are no routing conflicts at any of the switch elements? What speed advantage is needed to get the maximum possible throughput, under uniform random traffic?