Do your own work. Be neat and concise.

1. The figure below (similar to the figures on pages 3-23 and 3-24 of the lecture notes) shows the state of a simple crossbar scheduler at the start of a scheduling operation. Show the state of the controller after each step of the algorithm, until all possible matches have been made. Be sure to show how the pointers are updated. Do this for both the round robin algorithm and the i-SLIP algorithm.

2. The figure below (similar to the figure on page 3-31 of the lecture notes) shows the initial state of a CCF crossbar scheduler at the start of a scheduling operation. Show the state of the controller after each step of the algorithm.

<table>
<thead>
<tr>
<th>new arrivals</th>
<th>initial state</th>
</tr>
</thead>
<tbody>
<tr>
<td>b8</td>
<td>c4 b3</td>
</tr>
<tr>
<td>h8</td>
<td>a3 e6</td>
</tr>
<tr>
<td>c8</td>
<td>b7 h4 d3</td>
</tr>
<tr>
<td>a8</td>
<td>e7</td>
</tr>
<tr>
<td>d8</td>
<td>d6 a5 h5</td>
</tr>
<tr>
<td>b8</td>
<td>a3 b2</td>
</tr>
</tbody>
</table>

- 1 -
3. (20 points) In one variant of the LOOFA crossbar scheduling algorithm, outputs that receive multiple "bids" from inputs, select inputs based on the timestamps of the contending cells (cells with small timestamps are preferred over cells with larger timestamps). Also, cells are forwarded from the output-side queues in timestamp order. Find a traffic pattern that demonstrates that even with a speedup of 2, this version of the LOOFA scheduling algorithm does not always forward cells in FIFO order.

4. Show that the stable matching algorithm described on page 3-30 does in fact produce a stable matching.

5. (50 points) On the web site, you will find a program xbStressPIM that was used to produce the results on pages 3-21 and 3-22 of the lecture notes. Create a new version of this program that implements the i-SLIP scheduling algorithm, instead of the random matching algorithm used in the original. Your program should allow the number of iterations to be limited to a specified number. Use your program to produce charts like those on 3-21 and 3-22 for i-SLIP. Provide four sets of charts: the first set should limit the algorithm to one iteration, the second set should limit it to two iterations, the third to four and the last set should not limit the number of iterations. Compare your charts to the ones in the notes. Explain any differences that you observe.

6. One way to implement multicast in a crossbar switch with VOQs is for the IPPs to copy each arriving multicast cell to the VOQs for all outputs that are to receive copies. With this approach, the multicast cells appear no different than unicast cells to the crossbar. Show that a crossbar can forward multicast cells with a fanout of $F$ in a work-conserving fashion, using the LOOFA algorithm and a speedup of $F+1$.

7. Consider a multistage network in which each switch element has four inputs and four outputs, and there is an eight slot output queue associated with each output. Suppose the four queues contain 2, 5, 6 and 7 cells, respectively. If the system uses grant flow control, how many grants can be sent to the upstream neighbors, at the start of the cell cycle? Why? Suppose the system uses acknowledgement flow control and each of the upstream neighbors sends a cell to the given switch element. What is the maximum number of cells that may be acknowledged by the switch element in this situation? Why? What is the minimum number? Why?

8. The paper by Choudhury and Hahne (available on the web site) describes a method for regulating the use of the memory space in a shared buffer switch element. Consider the following scenario for a d port switch element with a shared buffer that has space for 8 cells. At time 0, all input links have a load of 100%, with all the arriving cells directed to output 0. This arrival pattern continues until the queue for output 0 reaches the maximum length allowed by the buffer sharing mechanism. At this point, the arriving traffic changes, so that
all arriving cells are directed to output 1. For what values of the dynamic sharing parameter \( \alpha \), will the length of the queue for output 1 match the length of the queue for output 0, before any cells going to output 1 get discarded?

9. Give the values of \( \tau_{5,2}(12) \) and \( \tau_{9,4}(17) \).

Draw a picture of the network defined by the expression \( X_{2,2} \otimes (X_{2,2} \times X_{3,3}) \otimes X_{2,3} \).

How many paths are there between an input \( x \) of this network, and an output \( y \)?

Suppose this network is used to implement a cell switch that uses dynamic routing and balances the load evenly across all available paths in the network. Suppose there is a virtual circuit with a bandwidth of 100 Mb/s from input 4 to output 11. On your diagram, highlight all the links that carry traffic from this virtual circuit and label them with the amount of bandwidth used on each line.

10. Consider the network \( K_{n,d,h} \) defined by

\[
K_{n,d,0} = D_{n,d} \quad K_{n,d,h} = X_{d,d} \otimes K_{n/d,d,h-1} \otimes X_{d,d} \quad \text{for } h > 0
\]

Show that \( K_{n,d,h} = D_{n,d,h}^* \) by induction, using the associativity of the parallel connection operation.