1. On the website, you will find a spreadsheet (qeval.xls) that implements the iterative computation described on page 3 of the lecture notes. Use this to produce a chart similar to the one on page 4 of the notes. In your case, let the initial value of $\lambda$ be .4. Then, at time 10, change it to .1, and at time 20 change it to .25. Extend the time range to 50 steps.
2. If we take the continuous time version of the simply queueing model discussed in the notes, and let $B$ become infinite, we get the M/M/1 queueing model. Derive an expression for the probability distribution of the waiting time for an M/M/1 queue. Note that if the queue has length $k$ when a packet arrives, then the waiting time for that packet is the sum of $k$ independent exponential random variables with mean value equal to the average time it takes to transmit a packet on the link.

3. The probability that an M/M/1 queue has exactly $k$ packets is $(1-\rho)^k$. If we let $1/\mu$ be the mean time to send a packet on the link, then the time to send $k$ packets is given by an Erlang distribution with density function $(\mu(\mu t)^{k-1}/(k-1)!))e^{-\mu t}$. So, for $t>0$, the waiting time distribution is

$$f_w(t) = \sum_{k \geq 1} (1-\rho)^k \frac{\mu(\mu t)^{k-1}}{(k-1)!} e^{-\mu t} = (1-\rho)e^{-\mu t} \sum_{k \geq 1} (\frac{\lambda}{\mu})^k \frac{\mu(\mu t)^{k-1}}{(k-1)!}$$

$$= \lambda(1-\rho)e^{-\mu t} \sum_{k \geq 1} (\frac{\lambda t}{\mu})^{k-1} = \lambda(1-\rho)e^{-\mu t}$$

for $t=0, f_w(t)=(1-\rho)$.

4. Consider a variation of the basic queueing model (discrete time version), in which the arriving packets consist of high priority packets and low priority packets. An arriving high priority packet is added to the queue, if there room in the queue, but a low priority packet is added only if the number of packets currently in the queue is less than or equal to a threshold value. Draw a finite state model for such a queue, assuming the queue has space for a total of four packets and that the threshold is two. Label all the edges with the appropriate transition probabilities. Let the total arrival probability be $\lambda=\lambda_0+\lambda_1$, where $\lambda_0$ is the arrival probability for the high priority packets and $\lambda_1$ is the arrival probability for the low priority packets (at most one packet arrives during each time step) and let the departure probability be $\mu$.

Write down any two of the steady-state balance equations for your finite state model

$$\pi(0) = \pi(0)(1-\lambda) + \pi(1)(1-\lambda)\mu$$

$$\pi(2) = \pi(1)\lambda(1-\mu) + \pi(2)(\lambda\mu + (1-\lambda)(1-\mu)) + \pi(3)(1-\lambda_0)\mu$$
5. The chart below shows the loss probability for a queue, when subjected to three different types of traffic. In all cases, the traffic is bursty and in all cases, the average burst lengths are the same and the buffer lengths are the same. Type A traffic has a burst transmission rate equal to \(0.2 \times \text{(link rate)}\) and has exponentially distributed burst lengths. Type B traffic has a burst transmission rate equal to \(0.2 \times \text{(link rate)}\) and Pareto distributed burst lengths, with a shape parameter of 1.4. Type C traffic has a burst transmission rate equal to \(0.05 \times \text{(link rate)}\) and has exponentially distributed burst lengths. Label the three curves, to show which curve corresponds to which type of traffic. Explain why you labeled them as you did. What can you say about the ratio of the average burst length to the size of the buffer?

The type B traffic has a larger loss probability than the type A traffic because of the higher variability caused by the Pareto distribution. On the other hand, type C traffic has a smaller loss probability than the type A traffic because it has a smaller burst transmission rate. So, the middle curve is A, the top is B and the bottom is C.

When burst lengths are larger than the buffer size, there is little difference between results for the Pareto and exponential distributions. Since there is a significant difference here, the burst length must be smaller than the buffer size.