CSE 240
Logic and Discrete Mathematics

Instructor: Todd Sproull

Department of Computer Science and Engineering
Washington University in St. Louis

CSE 240 Course Information

- **Time and Location**
  - Louderman 458
  - Tuesday and Thursday 11:30AM – 1:00PM
- **Website**
  - http://research.engineering.wustl.edu/~todd/cse240
- **Instructor Email Address**
  - todd@wustl.edu
- **Book**
  - Discrete Mathematics and Its Applications by Rosen 7th edition
- **Office Hours**
  - By Appointment
- **Head TA**
  - Emily Feng
- **TAs**
  - Daniel Munro
  - Anna Gautier
  - Makai Mann
  - Max Lyons
  - Emily Coco
  - Ke Xu
  - Gwynthel Pearson
  - Michael Wang
  - Cindy Le
  - Alexander Durgin
  - Trang Nguyen
  - Younis Mahmoud
  - Shitianyu Pan

- **What is this course about?**
  - An introduction to mathematical fundamentals needed by a Computer Scientist
  - Why is mathematics needed?
Grading

- **Three in class exams**
  - 18% of total grade per exam (54% total)

- **~10 Homework Assignments**
  - 36% of total grade
  - Lowest score dropped

- **~10 Short Quizzes and Group Assignments**
  - 10% of total grade
  - Lowest score dropped

- **No Final Exam**

Some of the Topics Covered

- **Logic**
  - Is this statement True or False?

- **Proofs**
  - Can you prove that your claim is True?
  - Direct, Indirection, and Proofs by Contradiction
  - Mathematical Induction

- **Counting**
  - How many ways can I pick $n$ items from a set of $m$?
  - How do I win at Blackjack?

- **Probability**
  - Existence proofs
  - Expected value, variance...

- **Graphs**
  - Can we model a collection of computers connected together?

- **Finite State Machines**
  - Can a computer carry out this task?
  - How do we model computation?
Reading Assignment

• Read Sections 1.1 - 1.3 by Thursday

Section 1.1 Propositional Logic

• Proposition
  – A declarative statement that is either true or false

• Examples of propositions
  – $1 + 2 = 3$
  – $1 + 2 = 4$
  – Three is odd
  – Four is odd

• Not propositions
  – Funny
  – Apple
  – What time is it?

• Typically represented with the letters p, q, r, and s
  – Suppose p ...
Propositional Logic Negation

- Suppose \( p \) is a proposition
  - The negation of \( p \) is \( \neg p \)
    - "It is not the case that \( p \)"
    - Also represent by \( \bar{p} \)

- Example
  - Three is NOT odd
  - Four is NOT odd
  - March does NOT have 31 days

- Negate the following
  - At least 10 inches of rain fell today

- Answer
  - It is not the case that at least 10 inches of rain fell today
  - Less than 10 inches of rain fell today

- Truth table for negation:

<table>
<thead>
<tr>
<th>( p )</th>
<th>( \neg p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Conjunction

- A conjunction is equivalent to using the word “AND”
  - Denoted by the \( \land \) symbol
  - Suppose \( p \) and \( q \) are propositions
  - \( p \land q \)

- A conjunction is TRUE when both \( p \) and \( q \) are TRUE

- Example
  - This class is fun AND I like pizza

- Truth Table for conjunction:

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p \land q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Disjunction

- A disjunction is equivalent to the word “OR"
  - Denoted by the symbol $\lor$
  - $p \lor q$

- A disjunction is false when both $p$ and $q$ are false

- Example
  - Today is Friday OR it is raining today

- Truth table for Disjunction

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>$p \lor q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Exclusive Or

- The exclusive or is true when either $p$ or $q$ is TRUE
  - But not both $p$ and $q$
  - Denoted by the symbol $\oplus$

- Example
  - I will EITHER pay attention in class OR fall asleep

- Truth Table for exclusive or

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>$p \oplus q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Implications

- **Denoted by the symbol →**
  - $p \rightarrow q$ corresponds to “If $p$, then $q$” or “$p$ implies $q$” or “$q$ whenever $p$”

- **Example**
  - If I work hard in this class, then I will earn an A in CSE240
  - If today is Friday, then $2 + 3 = 5$ (True or False? Why?)
  - If today is Friday, then $2 + 3 = 6$ (True or False? Why)

- **Truth Table for implications**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>$p \rightarrow q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>?</td>
</tr>
</tbody>
</table>

Truth table for $p \rightarrow q$

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>$p \rightarrow q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
More about $p \rightarrow q$

- Different English representations of $\rightarrow$
  - $p$ implies $q$
  - $q$ whenever $p$

- $x$ is odd $\rightarrow x + 1$ is even
  - If $x$ is odd, it is true
  - If $x$ is NOT odd, the whole statement is true

- Propositions can only be True or False
  - No undefined

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \rightarrow q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$T$</td>
<td>$F$</td>
<td>$F$</td>
</tr>
<tr>
<td>$F$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$F$</td>
<td>$F$</td>
<td>$T$</td>
</tr>
</tbody>
</table>

Logical Equivalence

- Denoted by the symbol $\iff$
  - $p \iff \neg \neg p$ corresponds to “$p$ is logically equivalent to NOT NOT $p$”

- Consider $\neg p \lor q \iff p \rightarrow q$
  - Are they logically equivalent?
  - How can we prove that?

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$\neg p$</th>
<th>$\neg p \lor q$</th>
<th>$p \rightarrow q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$F$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$T$</td>
<td>$F$</td>
<td>$F$</td>
<td>$F$</td>
<td>$F$</td>
</tr>
<tr>
<td>$F$</td>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$F$</td>
<td>$F$</td>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
</tbody>
</table>
Converse, Contrapositive, and Inverse

- Formed as variants of the conditional statement $p \rightarrow q$
- **Converse**
  - $q \rightarrow p$
- **Contrapositive**
  - $\neg q \rightarrow \neg p$
- **Inverse**
  - $\neg p \rightarrow \neg q$

**Examples**
- Consider the conditional statement
  - “The St. Louis Cardinals win whenever it is raining”
  - “If it is raining, then the Cardinals win”
  - **Converse**
    - If the Cardinals win, then it is raining
  - **Contrapositive**
    - If the Cardinals do not win, then it is not raining
  - **Inverse**
    - If it is not raining, then the Cardinals do not win

Biconditional Statement

- Denoted by the symbol $\iff$
- $p \iff q$ corresponds to “$p$ if and only if $q$”
  - or “$p$ is necessary and sufficient for $q$”
  - or “$p$ iff $q$”

- $p \iff q$ is true only when $p$ and $q$ have the same truth values

**Example**
- You can take the flight if and only if you buy a ticket

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>$p \iff q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

**Truth Table for biconditional statement**
Compound Propositions

- We are able to combine multiple propositions together to build more complicated propositions.

- Construct a truth table for the following proposition:
  - \((p \land q) \rightarrow (\neg p \lor q)\)

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>q</td>
<td>\neg p</td>
<td>p \land q</td>
<td>\neg p \lor q</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Logic and Bit Operations

- Computers represent information using bits
  - A bit is a symbol with values 0 and 1

- Boolean Variables use values of True or False
  - Boolean variables can be represented with a bit

- Logical operations can be performed by replacing T and F with 1 and 0
  - Substitute \(\wedge\), \(\lor\), and \(\oplus\) with AND, OR, and XOR
  - Example: \(0 \lor 1 = 1\)

- Bit Strings are a sequence of 0 or more bits
  - length is string is the number of bits

- Able to perform bitwise operations on bit strings

- Example

<table>
<thead>
<tr>
<th>Bitwise OR</th>
<th>Bitwise AND</th>
<th>Bitwise XOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>111 11101</td>
<td>001 00001</td>
<td>110 11100</td>
</tr>
</tbody>
</table>

I need some help with my math.
Group Exercise